Quiz #13; Wed, 4/27/2016

Math 53 with Prof. Stankova

Section 110; MWF12-1

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Problem. Let $\mathbf{F} := \langle P, Q \rangle$ a vector field defined on $\mathbb{R}^2 \setminus \{(0,0)\}$ where

$$P = \frac{-2xy}{(x^2+y^2)^2} \quad \text{and} \quad Q = \frac{x^2-y^2}{(x^2+y^2)^2}$$

(a) Show that $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$. Say why this doesn't necessarily mean that $\mathbf{F}$ is not conservative on $\mathbb{R}^2 \setminus \{(0,0)\}$.

(b) Use the function $f(x,y) = \frac{y}{x^2+y^2}$ to show that $\mathbf{F}$ is in fact conservative on $\mathbb{R}^2 \setminus \{(0,0)\}$.

Solution. (a) By quotient rule, one computes that $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} = \frac{-2x^3+6xy^2}{(x^2+y^2)^3}$. Now, since the domain over which $\mathbf{F}$ is defined over is not simply connected, it may not be conservative.

(b) One checks that in fact, $\nabla f = \mathbf{F}$. So, $\mathbf{F}$ is conservative.