Combinatorial Optimization

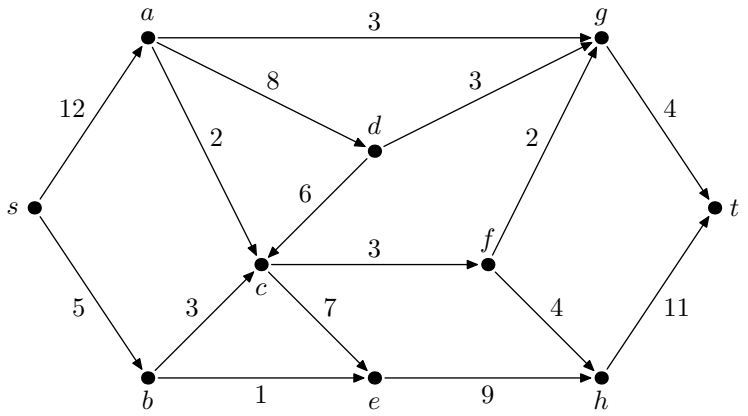
Problem set 6

Assigned Monday, June 22, 2015. Due Thursday, June 25, 2015.

1. Suppose a simple undirected graph has more than one minimum spanning tree. Can Prim's algorithm (or Kruskal's algorithm) be used to find all of them? Explain why or why not, and give an example.

2. Explain how a minimum spanning tree algorithm can easily be used to find a *maximum* spanning tree in a graph. Then explain why, if you want to find the *longest* path between two vertices in a graph, using this same technique with Dijkstra's algorithm does not work.

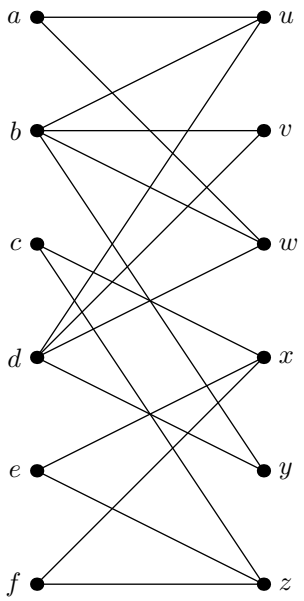
3. Find a maximum s–t flow and a minimum s–t cut in the following flow network.



4. Carefully describe an algorithm for the following problem: Given a simple undirected graph $G = (V, E)$, determine whether G is bipartite, and if so give a bipartition (i.e., a partition of the vertex set V into two sets U and W such that every edge has one endpoint in U and one endpoint in W). Illustrate the operation of your algorithm on two examples, one bipartite graph and one non-bipartite graph. Prove that your algorithm is correct in general.

[Hint: You may find it useful to prove and use Proposition 1 in Section A.2 of the textbook, on page 21.]

5. Find a maximum matching in the following bipartite graph.



6. A small trucking company has a fleet of five trucks, and on a certain day has seven loads to deliver. In the following tables, the capacities of the trucks and the sizes of the loads are both given in units of 1000 pounds.

Truck	Capacity	Daily cost
1	3	\$200
2	6	\$300
3	6	\$400
4	8	\$350
5	11	\$500

Load	Weight
A	1
B	2
C	3
D	4
E	4
F	5
G	8

The daily cost for a truck must be paid if that truck is to be used to make any deliveries. Because of their locations, loads A and D cannot be delivered by the same truck, nor can loads B and E. Formulate an integer program to determine which loads should be assigned to each truck in order to minimize the total daily cost.