Combinatorial Optimization

Midterm examination

Released Thursday, June 18, 2015. Due 1:30 p.m., Monday, June 22, 2015.

You must read, understand, and agree to follow all of the rules of this exam, which were given to you separately. By opening the envelope containing these problems, you have begun this exam and are bound by these rules until class on Monday. Here are the rules again as a reminder.

1. This exam consists of five problems. It is due at the beginning of class on Monday, June 22, 2015.
2. You may open the envelope containing the problems whenever you wish, but when you do so you are beginning the exam. From then until class on Monday, you are bound by these rules.
3. You may not discuss these problems or other topics from this course with anyone else after you begin the exam.
4. If you have questions during the exam, including questions about these rules, you may ask me (Brian) either in person or via e-mail. You may not ask anyone else, including Michael.
5. You may use the textbook, your course notes, your completed problem sets, and the resources on the course Web page. You may use Maple, the pivoting tool linked from the course Web page, and a calculator. You may not use any other external resources or tools.
6. Justify everything. The justification is often more important than the answer itself. If you use Maple, include a printout. If you can't figure out the answer, describe your thought process and what you have figured out; partial credit will be given. Clearly state any assumptions you make.

After completing this exam, please sign the statement accompanying the rules and submit it with your solutions.

Each of the five problems on this exam is worth 20 points.

1. Solve the following linear program. You may not use Maple for this problem.

$$
\begin{array}{rrcl}
\text{minimize} & 3x_1 + 2x_2 - x_3 - 4x_4 & & \\
\text{subject to} & x_1 + x_2 + x_3 + x_4 & = & 10 \\
& 2x_1 + x_3 - 2x_4 & \geq & 6 \\
& x_1 + 3x_4 & \leq & 30 \\
& 3x_1 + x_2 & \geq & -8 \\
& x_2 + 3x_3 & \geq & 3 \\
\end{array}
$$

$$x_1 \geq 0, \quad x_2 \text{ unrestricted}, \quad x_3 \leq 0, \quad x_4 \geq 0.$$

2. Formulate a linear program for the following scenario. Then solve the linear program and interpret your results.

> Leisure Furniture, Inc. (LFI) makes outdoor chairs, tables, lounges, and benches. The main resources required for production are plastic webbing, metal tubing, and wood. The number of units of each resource required per item and the per-item profit are given below.

	Tubing (feet)	Webbing (yards)	Wood (board feet)	Per-unit profit
Chair	18	28		\$11
Table	18		10	\$18
Lounge	24	35		\$14
Bench	20		14	\$ 9

The main activities in production are tube bending and assembly. The number of hours of each activity required per unit of each product are given in the table below.

	Chair	Table	Lounge	Bench
Tube bending	1/3	1/8	1/4	1/8
Assembly	1/2	1/2	1/3	1/3

LFI is planning its first week's production for the spring. Available during that week are 120 hours of bending time, 160 hours of assembly time, 6,000 feet of tubing, 5,000 yards of webbing, and 1,000 board feet of wood. The lounge is the most popular product, so LFI wants to make at least 100 of them. Tables and chairs are commonly sold in sets of two chairs and a table, so they want to make at least twice as many chairs as tables. Otherwise they want to make the products that will maximize their profit if sold.

3. Find an extreme point of the feasible region of the following linear program.

$$\begin{aligned} \text{maximize} \quad & x_1 - 2x_2 + x_3 \\ \text{subject to} \quad & 3x_1 + 3x_2 - x_3 - 2x_4 \le 90 \\ & x_1 + 5x_3 \ge 20 \\ & -4x_1 - x_2 + x_3 - x_4 = 6 \\ & x_2 - 2x_3 + x_4 \le 40 \\ & x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0, \quad x_4 \ge 0. \end{aligned}$$

4. Consider a project consisting of several activities, each having a usual time and a set of immediate prerequisites. Some of the activities can be sped up; these activities additionally have a crash time and a per-unit speedup cost. (This is the kind of problem we considered when we discussed the critical path method.) Suppose that the overall project needs to be sped up to meet a deadline, and it is desired to do so at minimum total speedup cost. Is it true that only the critical activities (under the usual times) are candidates to be sped up? If so, explain why. If not, give a counterexample.

5. An important communications line is to be built between locations s and t in a dangerous, disaster-prone area. The line cannot be built directly; it will need to be built as a sequence of links joining intermediate points a, b, …, g. The following table shows the estimated probabilities of *failure* of a link built between pairs of points. (Not all pairs of points are possible locations for a communications link; impossible locations are indicated with a dash.) If any link fails, then the entire communications line fails. Determine a route for the line that minimizes the probability of failure.

	s	a	b	c	d	e	f	g	t
s	—	1.5%	—	—	—	—	3.0%	4.0%	—
a	1.5%	—	—	—	5.0%	—	2.0%	—	—
b	—	—	—	6.0%	8.0%	—	—	—	0.5%
c	—	—	6.0%	—	—	2.0%	3.5%	7.0%	—
d	—	5.0%	8.0%	—	—	—	2.5%	—	—
e	—	—	—	2.0%	—	—	—	5.5%	5.0%
f	3.0%	2.0%	—	3.5%	2.5%	—	—	1.0%	—
g	4.0%	—	—	7.0%	—	5.5%	1.0%	—	—
t	—	—	0.5%	—	—	5.0%	—	—	—