ILLUMINATING A ROOM

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INTRODUCTION

"Illuminating a room " is a very interesting topic as it uses variety of applications of calculus. This project is basically about finding the height at which the amount of illumination will be maximum . Part (A) explains about obtaining the height at which 1 light source should be placed to attain maximum amount of illumination to cover all 4 corners of the room. Part (B) is even more interesting as it tells us about finding the height at which 2 light sources should be placed to attain maximum amount of illumination .

PART (A) -

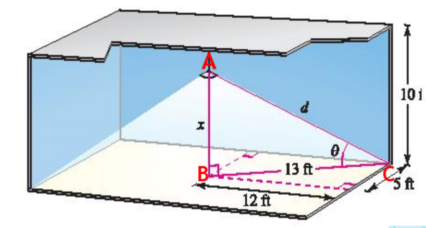
OBJECTIVE :

- We would like to maximize the amount of illumination by adjusting the height at which the light should be placed.
- Given: Amount of illumination(L) ∝ Intensity of light source (i)

  ∝ square of the light source (d^2)

  ∝ sin Θ where Θ is the angle at which light strikes

- Therefore, $l = \dfrac{k(i\sin(\theta))}{d^2}$, Where K is a constant



IDENTIFYING THE CONSTANT :

- Here 'k' and the intensity(i) are constant. Multiplying these constants will give us another constant that we named 'K'.
- In ABC from the figure is a right - angled triangle. Here sin Θ is the ratio of the opposite side to the hypotenuse.
- Therefore, sin Θ = opposite side/hypotenuse(x/d),where 'x' is the height at which the light is placed from the ground, is the hypotenuse.
- According to Pythagorean theorem,

  $(AB)^2+(BC)^2=(AC)^2$

  $x^2+169^2=d^2$

3. REDUCING THE OBJECT FUNCTION TO ONE VARIABLE:

Now we have Sin Θ and 'd' in terms of only one variable that is 'x'.

Sin Θ = x/d

d = root(x^2 +169)

k = K(i)

We also know that L = K(i (Sin Θ))/ (d)^2

Now we must substitute the value of sin theta and d in terms of x and K(i) as k in the above formula

Therefore , after substitution , we get :

L = k(x)/ (x^2 +169)^3/2

Now we have the objective function that is only in terms of x.

4. DOMAIN OF THE OBJECTIVE FUNCTION:

We know that 'x' in the objective function represents height ,so it cannot be negative. We also know that the maximum height at which the light source can be placed cannot be more than 10 feet from the ground

Therefore, 0 ≤ x ≤ 10 .

The domain of the function is x ∈ [0,10] .

5. DIFFERETIATING THE OBJECTIVE FUNCTION

We must now find the differentiate the object function in order to find the critical numbers.

Our objective function is L = kx/(x^2+169)^3/2

To differentiate this function , we use three different rules of differentiation.

They are the quotient rule, chain rule and the constant multiple rule.

If 'u' and 'v' are differentiable , then :

Quotient rule : d/dx(u(x)/v(x)) = v(x).d/dx(u(x)) - [u(x). d/dx(v(x))]/ [v(x)]^2

Constant multiple rule : d /dx (cu) = c.d/dx(u) [where c is a constant]

Chain rule: d/dx(u(v(x))) = u'(v(x)).v'(x)

Using the following rules, we can differentiate this function.

After differentiation , we get:

d/dx(L) = (k(x^2+169)^(1/2))(x^2+169-3x^2)/(x^2+169)^3

6.FINDING THE CRITICAL NUMBERS

Now we must equate d/dx(L) to zero or check if the derivative is undefined at any value of x.

(i) d/dx(L) = 0

(k(x^2+169)^(1/2))(x^2+169-3x^2)/(x^2+169)^3 = 0

After simplification , we get :

(k(x^2+169)^(1/2)) = 0	(x^2+169-3x^2) = 0
Finally , we get x = root(-169)	Finally , we get x = 13/root(2)
Here the value of x does not exist	

(ii) To check if the derivative is undefined for any value of x

If we clearly analyze the denominator of the function and equate it to 0 , we get x is equal to root(-169) which tells us that the value of x does not exist.

There is no value of x for which the derivative is undefined.

Therefore, we have only one critical number that is 13 / root(2)

7. TESTING THE CRITICAL NUMBERS

We must now find the value of L by plugging in the three values of 'x' that is 0,13/root(2) and 10.

X	L
0	L = 0
13/root(2) = 9.19	L = 0.00227
10	L = 0.00226

8. ANSWERING THE QUESTION

We find that the maximum value of illumination is when the light is placed at 9.19 feet from the ground.

From the above statement, we can conclude that the light source must be placed at a height of 9.19 feet so that the corners of the room receive as much light as possible.

PART (B) -

1. OBJECTIVE:

- We would like to maximize the amount of illumination by adjusting the height at which the light should be placed.
- Given: Amount of illumination(L) ∝ Intensity of light source (i)

  ∝ square of the light source (d^2)

  ∝ sin Θ where Θ is the angle at which light strikes

- Therefore, $l = \dfrac{k(i\sin(\theta))}{d^2}$, Where K is a constant

2. IDENTIFYING THE CONSTANTS:

- We know that

  $l = \dfrac{k(i\sin(\theta))}{d^2}$,

- Triangle ABC,DEC are right angled triangles.
- Here, sin Θ = x'/d'

  sin Θ = x'/d" ,where " x' " is the height of light from ground & " d_1" & " d_2" are hypotenuses

- According to Pythagorean theorem,

  $(AB)^2+(BC)^2=(AC)^2$

  $(x')^2+(9.43)^2=(d_1)^2$ --(1)

  $(DE)^2+(EC)^2=(DC)^2$

  $(x')^2+(16.8)^2=(d_2)^2$ --(2)

3. REDUCING THE OBJECT FUNCTION TO ONE VARIABLE:

- Now, we have $l_1 = \dfrac{k(i\sin(\theta))}{d_1^2}$, $l_2 = \dfrac{k(i\sin(\theta))}{d_2^2}$

Substituting d1 & d2 replacing (i sin (Θ)) as x' ,we get :

$L_1 = \dfrac{R\cdot x'}{((x')^2+88.92)^{3/2}}$ | $L_2 = \dfrac{R\cdot x'}{((x')^2+(282.24))^{3/2}}$

- Therefore,

  Total illumination(L) = $L_1 + L_2 = \dfrac{R\cdot x}{((x')^2+88.92)^{3/2}} + \dfrac{R\cdot x}{((x')^2+(282.24))^{3/2}}$

4. DOMAIN OF THE OBJECTIVE FUNCTION:

- We know that ' x' ' in the objective function represents height ,so it cannot be negative. We also know that the maximum height at which the light source can be placed cannot be more than 10 feet from the ground, Therefore, 0 ≤ x ≤ 10 .

  The domain of the function is x ∈ [0,10] .

5. DIFFERETIATING THE OBJECTICE FUNCTION:

- We must now find the differentiate the object function in order to find the critical numbers.
- (i) $L_1'(x) = \dfrac{d}{dx}\left(\dfrac{R\cdot x'}{((x')^2+88.92)^{3/2}}\right)$

Differentiating L_1 we get -

$$L_1'(x) = K\left(-\frac{3x^2}{(88.92+(x')^2)^{5/2}} + \frac{1}{(88.92+(x')^2)^{3/2}}\right)$$

(ii) Differentiating L_2 we get -

$$L_2 = \frac{R\cdot x'}{((x')^2+(282.24))^{3/2}}$$

6. FINDIING THE CRITICAL NUMBERS:

(i)Now ,we must equate d/dx(l_1+L_2) to zero if the derivative is undefined at any value of x.

$$L_2'(x) = K\left(-\frac{3x^2}{(282.24+(x')^2)^{5/2}} + \frac{1}{(282.24+(x')^2)^{3/2}}\right)$$

$$\Rightarrow \frac{d}{dx}(L_1+L_2) = 0$$

$$\frac{d}{dx}\left(\frac{Rx}{((x')^2+88.92)^{3/2}} + \frac{R\cdot x}{((x')^2+282.24)^{3/2}}\right) = 0$$

After simplification and solving for zero ,We get →

$$\therefore R=0 \quad (\text{or}) \quad \left(\frac{88.92-2x^2}{(x^2+88.92)^{5/2}} + \frac{282.24-2x^2}{(x^2+282.24)^{5/2}}\right) = 0$$

$\therefore x = 7.39668$ or -7.39668 → not possible

(ii) To check if the derivative is undefined for any value of x :

The value of x exceeds the range, so there is no possible value of 'x' that causes this derivative to be undefined.

7. TESTING THE CRITICAL NUMBERS :

We must now find the value of 'L_1 + L_2' by plugging in the value of 'x'.

x	L_1+L_2
0	L_1+L_2 = 0
7.39668	L_1+L_2 = 0.005493
10	L_1+L_2 = 0.005189

8.ANSWERING THE QUESTION :

We observe that this arrangement changes the height (≈7.39668 ft) at which the lights should be hung to best illuminate the corners of the floor.