

Analysis of a simplex tableau (ignoring degeneracy)

Is the corresponding solution **optimal**?

- If there is a negative number in the objective row: **No**, the corresponding solution is not optimal. (Increasing the value of the corresponding variable will increase the objective value.)

In this case: Is the problem **unbounded**?

- If any negative entry in the objective row has no positive numbers above it: **Yes**, the problem is unbounded. (The value of the corresponding variable can be made arbitrarily large in such a way that no constraints are violated, and this will make the objective value arbitrarily large as well.)
- If all negative entries in the objective row have at least one positive number above them: **No conclusion**. (The objective value can be increased by pivoting on a column with a negative entry in the objective row. This will lead to a new tableau. Then analyze the new tableau.)
- If all numbers in the objective row are nonnegative: **Yes**, the corresponding solution is optimal. (No variable can be increased to increase the objective value.)

In this case: Is this optimal solution **unique**?

- If all nonbasic columns have positive entries in the objective row: **Yes**, the optimal solution is unique. (Bringing any nonbasic variable into the basis will decrease the objective value.)
- If there is a nonbasic column with a zero in the objective row: **No**, the optimal solution is not unique. (The value of the corresponding variable can be increased to get a different optimal solution.)

In this case: Is there **another optimal basic solution**?

- If there is a nonbasic column with a zero in the objective row and at least one positive number above it: **Yes**, there is another optimal basic solution. (Pivoting on this column will produce it.)
- If none of the nonbasic columns having zero in the objective row have positive numbers above: **No**, there is no other optimal basic solution. (But there are still infinitely many optimal solutions—the set of optimal solutions forms an infinite ray extending from the optimal basic solution.)