Whiz-Bang Manufacturing produces four lines of products. Each unit of each of the product lines requires one part called a hypergizmo, and Whiz-Bang has 270 hypergizmos on hand. Each unit of each of the product lines (except the first) also requires one or more parts called e-sprockets; Whiz-Bang has 480 e-sprockets in stock. This week Whiz-Bang has 600 hours of labor available for assembly and, separately, 566 hours available for finishing. The per-unit profit and resource requirements for each of the product lines are given in the table below:

Product line	Profit	E-sprockets	Assembly hours	Finishing hours
1	$10	0	2	1
2	$14	1	2	2
3	$17	3	5	3
4	$12	2	3	2

The marketing department has asked for at least 30 units of the first product line to be produced this week, because that is the original Whiz-Bang product and they are planning a "throwback" marketing campaign. How much of each product line should Whiz-Bang produce this week in order to maximize total profit?

Here is the linear program:

$$\begin{array}{lrrrrl} \text{maximize} & 10x_1 + 14x_2 + 17x_3 + 12x_4 & & & & \text{[profit]} \\ \text{subject to} & x_1 + x_2 + x_3 + x_4 \leq 270 & & & & \text{[hypergizmos]} \\ & x_2 + 3x_3 + 2x_4 \leq 480 & & & & \text{[e-sprockets]} \\ & 2x_1 + 2x_2 + 5x_3 + 3x_4 \leq 600 & & & & \text{[assembly hours]} \\ & x_1 + 2x_2 + 3x_3 + 2x_4 \leq 566 & & & & \text{[finishing hours]} \\ & x_1 \qquad\qquad\qquad\qquad \geq 30 & & & & \text{[production requirement]} \\ & x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0. & & & & \end{array}$$

Here is the initial simplex tableau:

x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	a_5	
1	1	1	1	1	0	0	0	0	0	270
0	1	3	2	0	1	0	0	0	0	480
2	2	5	3	0	0	1	0	0	0	600
1	2	3	2	0	0	0	1	0	0	566
1	0	0	0	0	0	0	0	−1	1	30
−10	−14	−17	−12	0	0	0	0	0	0	0
−1	0	0	0	0	0	0	0	1	0	−30

And here is the optimal simplex tableau:

x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	a_5	
0	1	0	2/3	5/3	0	−1/3	0	1	−1	220
0	0	0	1/3	1/3	1	−2/3	0	−1	1	200
0	0	1	1/3	−2/3	0	1/3	0	0	0	20
0	0	0	−1/3	−4/3	0	−1/3	1	−1	1	36
1	0	0	0	0	0	0	0	−1	1	30
0	0	0	3	12	0	1	0	4	−4	3720

1. What is the optimal basic solution for the original linear program?

2. A hypergizmo salesperson contacts Whiz-Bang and offers to sell them some more hypergizmos. How much should Whiz-Bang be willing to pay per unit for more hypergizmos? How many should they be willing to buy at that price?

3. Suppose an employee were to open a dusty closet and discover a long-forgotten box containing a dozen hypergizmos. How would this windfall change Whiz-Bang's total profit and production schedule?

4. A competitor, Kablooie Industries, has cut the selling price of one of their product lines, and in response the Whiz-Bang sales department would like to reduce the selling price (and hence the profit) of their second product line by $1 per unit, effective immediately. What effect would this change have on Whiz-Bang's total profit and production schedule?

5. Whiz-Bang receives an advertisement from E. Spacely E-Sprockets, Inc., announcing a special sale on e-sprockets this week only! How much should Whiz-Bang be willing to pay per unit for more e-sprockets for use in this week's production?

6. The sales department is considering a change in the selling price of the first product line. What range of changes to the selling price for this product line will leave the current optimal basis unchanged?

7. The sales department would like to increase the number of the fourth product line sold. Assuming that Whiz-Bang can sell all of the products it makes, what change to the selling price would be necessary to make it profitable to do so?

8. If Whiz-Bang has the opportunity to hire more labor, should it hire more labor for assembly or for finishing? How much should it be willing to pay for each hour of this labor? How many additional hours of labor should it be willing to hire at this rate?

9. What range of per-unit profit for the third product line will leave the optimal basis unchanged?

10. What is the effect of the marketing department's request?

Answers

1. $x_1 = 30$, $x_2 = 220$, $x_3 = 20$, $x_4 = 0$.
2. Whiz-Bang should be willing to buy up to 27 more hypergizmos at up to $12 each.
3. With twelve additional hypergizmos, Whiz-Bang should produce 30, 240, 12, and 0 units of the four product lines, respectively. Their total profit will increase to $3,864.
4. If the per-unit profit on the second product line is reduced by $1, the optimal production schedule remains the same (30, 220, 20, and 0 units of the four product lines, respectively), but total profit will be reduced to $3,500.
5. Whiz-Bang should not be willing to buy additional e-sprockets at any (positive) price for use in this week's production.
6. The selling price of the first product could be increased by $4 or decreased by any amount without changing the optimal basis.
7. The selling price of the fourth product must be increased by at least $3 before it becomes profitable to produce and sell.
8. Whiz-Bang should not be willing to hire more labor for finishing. It should be willing to hire up to 108 additional hours of assembly labor at up to $1 per hour (if it can possibly find labor that cheap—maybe Oompa-Loompas).
9. The per-unit profit for the third product line can range from $14 to $35 without changing the optimal basis.
10. The total effect of the marketing department's request is to decrease total profit by $120; the optimal total profit could be $3,840 if that request were ignored.