21-110: Problem Solving in Recreational Mathematics

Spring break puzzles

Wednesday, March 3, 2010

Problem 1. ("Threaded Pins," *Thinking Mathematically*, page 52.)

Problem 2. ("Leapfrogs," *Thinking Mathematically*, page 57.)

Problem 3. ("Cartesian Chase," *Thinking Mathematically*, page 162.) This is a game for two players on a rectangular grid with a fixed number of rows and columns. Play begins in the bottom left-hand square where the first player puts his mark. On his turn a player may put his mark into a square

directly above
or directly to the right of
or diagonally above and to the right of

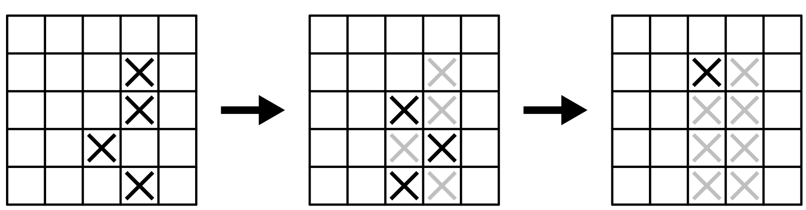
the last mark made by his opponent. Play continues in this fashion, and the winner is the player who gets their mark in the upper right-hand corner first. Find a way of winning which your great aunt Maud could understand and use.

Problem 4. ("Thirty-one," *Thinking Mathematically*, page 197.) Two players alternately name a number from 1, 2, 3, 4, 5. The first player to bring the combined total of all the numbers announced to thirty-one wins. What is the best number to announce if you go first?

Problem 5. You have an $n \times n$ chessboard and a marker. You begin by marking an X through some of the squares of the chessboard. Next, you mark an X through all of the squares which share at least two sides with squares already containing an X. You continue marking an X through such squares until there are no more unmarked squares that share at least two sides with marked squares. (Note that adding an X may open the possibility for another X to be added.) Sometimes when you do this, every square on the board will be marked with an X. Other times, the process will stop with some empty squares left over.

See the picture below for the process on a 5×5 chessboard with a certain four squares initially marked. In the end, a total of eight squares are marked.

In terms of n, what is the minimum number of marked squares you can start with and still end up with every square on the board marked with an X?



Problem 6. (From `http://stanwagon.com/wagon/Misc/bestpuzzles.html`.) A tunnel underneath a large mountain range serves as a conduit for 1001 identical wires; thus, at each end of the conduit, one sees 1001 wire-ends. Your job is to label all the ends with labels (#1, #2, ..., #1001), so that each wire has the same label at its two ends.

You may join together arbitrary groups of wires at either end; they will then conduct electricity through the join. Then you cross the mountains by a very expensive and dangerous helicopter ride to the other end, where you can feed electricity through any wire and check which of the other ends are live, attach notes to the wires, and make (or unmake) connections as desired. Then you fly back to the near end, perform the same sort of operations, fly back, and so on as often as required.

How can you accomplish your task with the smallest number of helicopter flights?

Problem 7. (From `http://stanwagon.com/wagon/Misc/bestpuzzles.html`.) Suppose we wish to know which windows in a 36-story building are safe to drop eggs from, and which will cause the eggs to break on landing. We make a few assumptions:

- An egg that survives a fall can be used again.
- A broken egg must be discarded.
- The effect of a fall is the same for all eggs.
- If an egg breaks when dropped, then it would break if dropped from a higher window.
- If an egg survives a fall, then it would survive a shorter fall.
- It is not ruled out that the first-floor windows break eggs, nor is it ruled out that the 36th-floor windows do not cause an egg to break.

If only one egg is available and we wish to be sure of obtaining the right result, the experiment can be carried out in only one way. Drop the egg from the first-floor window; if it survives, drop it from the second-floor window. Continue upward until it breaks. In the worst case, this method might require 36 droppings. Suppose two eggs are available. What is the least number of egg-droppings that is guaranteed to work in all cases?

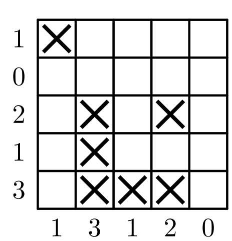
Problem 8. ("Pancakes," *Thinking Mathematically*, page 185.) When I make pancakes they all come out different sizes. I pile them up on a plate in the warming oven as they are cooked, but to serve them attractively I would like to arrange them in order with the smallest on top. The only sensible move is to flip over the topmost ones (i.e., insert a pancake flipper somewhere in the pile and flip over the smaller stack of pancakes above the flipper). Can I repeat this sort of move and get them all in order?

[Extension: Suppose I always burn one side of the pancakes when I cook them, and when I first pile them on the plate I place them burned side down. Can I sort them in the same way while guaranteeing that the burned sides will all be facing down when I am done?]

Problem 9. ("Desert Crossing," *Thinking Mathematically*, page 165.) It takes nine days to cross a desert. A man must deliver a message to the other side, where no food is available, and then return. One man can carry enough food to last for 12 days. Food may be buried and collected on the way back. There are two men ready to set out. How quickly can the message be delivered with neither man going short of food?

Problem 10. (Perfect square game.) This is a game for two players. Start with a given number (say, 10). On a player's turn, she finds the greatest perfect square that is no larger than the current number, and either adds or subtracts this perfect square to the current number to obtain a new number. The first player to reach 0 wins. What is the best strategy? (First of all, is it clear that the game will even end?)

Problem 11. (Grid sums.) This is another game for two players. The game is played on an $n \times n$ square grid, with markers on some squares initially. (The way in which these first markers are laid out is arbitrary; choose an interesting initial configuration and play with that.) Each player's turn consists of placing one marker on an empty square. The winner is the person who places the marker that first causes the sequence of row sums (the numbers of markers in each row) to be equal to the sequence of column sums, when both are sorted numerically. For example, the following configuration is a winning configuration, because both the sequence of row sums and the sequence of column sums, when sorted, are 0, 1, 1, 2, 3.



How should you play this game?