21-110: Problem Solving in Recreational Mathematics

Homework assignment 2

Assigned Monday, January 25, 2010. Due Monday, February 1, 2010.

Work at least **FOUR** of the following problems. All problems are of equal weight. If you submit solutions for more than four problems, you will get credit for your best four.

You are welcome to work with other students, but the solutions you hand in should be written in your own words. You are not allowed to see the paper another student is going to hand in. If you do collaborate with other students, list their names. If you use other sources, cite them. Give credit where credit is due. See the syllabus for more information about academic integrity.

Hints are encrypted with a *Caesar cipher*, in which each letter is replaced by the letter three places ahead in the alphabet, wrapping around to the beginning if necessary. For example, the letter A is encrypted as D, and Y is encrypted as B. To decrypt the hints, move each letter backward three places.

Problem 1. Suppose you have one copy of each free hexomino. Prove that it is impossible to arrange them into a rectangle.

Hint: Wkhuh duh wkluwb-ilyh iuhh kharplqrhv lq doo. Zkdw lv wkhlu wrwdo duhd? Zkdw glphqvlrqv frxog d uhfwdqjoh zlwk wklv duhd kdyh? Wub frorulqj wkh vtxduhv ri wkh kharplqrhv dv li wkhb zhuh fxw rxw ri d fkhfnhuerdug, dqg wkhq fodvvliblqj wkh kharplqrhv lqwr wzr jurxsv edvhg rq wklv frorulqj.

Problem 2. There are exactly 11 free hexominoes that can be folded to make a cube. These hexominoes are called *nets* of the cube.

(a) Draw the 11 nets of the cube.

(b) Explain why each of the other free hexominoes is not a net of the cube.

Hint: Rqh hasodqdwlrq pdb dssob wr vhyhudo kharplqrhv. Fryhu wkh hdvlhu fdvhv iluvw.

(c) All of the nets of the cube have the same perimeter. What is this common perimeter? Why is it not the same as the number of edges of a cube?

Problem 3. The following questions refer to two-dimensional shapes in the plane.

(a) Draw a shape that has reflection symmetry but not rotation symmetry. Show the line of symmetry.

(b) Draw a shape that has rotation symmetry but not reflection symmetry. Describe the rotation symmetry (how many degrees do you need to rotate the shape in order to have it look the same?).

(c) Draw a shape that has neither reflection symmetry nor rotation symmetry. Explain why this shape has neither of these symmetries.

(d) Draw a shape that has both reflection symmetry and rotation symmetry. Show the line of symmetry and describe the rotation symmetry.

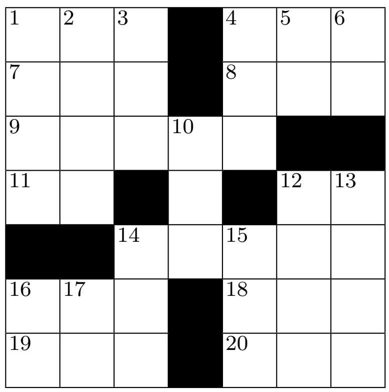
(e) Is there a shape that has *two* lines of symmetry but does *not* have rotation symmetry? If so, give an example. If not, why not?

Problem 4. Consider all of the free polyominoes up to area 6 (that is, the monomino, the domino, the trominoes, the tetrominoes, the pentominoes, and the hexominoes). Classify them according to their symmetry.

Hint: Wklqn derxw erwk uhiohfwlrq dqg urwdwlrq vbpphwub.

Problem 5. Design at least two interesting tessellations, based on different fundamental tilings. (For example, one may be based on the regular tiling of squares, and the other may be based on a semiregular tiling of hexagons and triangles.) Present your tessellations carefully—neatness is important. I suggest cutting out a template from heavy paper or cardboard to trace, or cutting out many individual tiles to paste together. Use color to highlight the repeating nature of your tessellations and explain any other interesting features. Be creative!

Problem 6. Solve the following crossword. Every numbered word in the puzzle must appear in the given word list, and no word may be used more than once. Explain the process you used.



Word list: an, as, at, do, eh, in, it, lo, me, no, oh, so, to; ago, aim, ale, all, and, ant, ash, ate, awe, din, dot, elk, end, has, hat, ice, ink, key, kid, lie, lot, mow, new, not, ode, owe, pat, pin, set, ski, spa, tag, tea, toe, ton, vat, wed, who, win, won, woo, zap, zip; also, apes, hall, have, head, iris, iron, noon, peas, soot, tend, urns, used, vase, zone; adore, aside, astir, hooks, legal, shark, shear, snort, stare, stars, start, syrup, voted.

Problem 7. Indiana Jones has made it through the deadly traps of an ancient temple and has discovered ten treasures inside. Unfortunately, his knapsack is too small to carry them all, so he must choose (wisely). He has made the following estimates of the objects' weights and values. If his knapsack can hold at most 20 pounds of treasure, which objects should he take to maximize the value of his loot?

Treasure	Weight	Value
Crown of Atahualpa	4 lb.	$ 4,000
Itzcoatl's Orb	6 lb.	9,000
Tablet of the Heavens	7 lb.	7,000
Golden Quetzal	13 lb.	17,000
Mask of the Ancients	5 lb.	5,000

Treasure	Weight	Value
Key of Silver Light	2 lb.	$ 2,000
Idol of Inti	10 lb.	15,000
Eternal Quipu	5 lb.	8,000
Goblet of Uxmal	3 lb.	4,000
Sacred Stone of Cuzco	8 lb.	11,000

Hint: Eh fduhixo, dqg gr qrw pdnh xqirxqghg dvvxpswlrqv. Frqvlghu doo srvvlelolwlhv. Lw pdb khos wr frqvlghu vpdoohu nqdsvdfnv. Iru hadpsoh, zkdw zrxog eh wkh ehvw zdb wr iloo d nqdsvdfn wkdw fdq krog whq srxqgv?

Problem 8. ("Cubes Cubed," *Thinking Mathematically*, page 36.) I have eight cubes. Two of them are painted red, two white, two blue and two yellow but otherwise they are indistinguishable. I wish to assemble them into one large cube with each colour appearing on each face. In how many ways can I assemble the cube?

Hint: Vhh wkh errn iru vrph vxjjhvwlrqv li brx jhw vwxfn. Rqh lpsruwdqw wklqj wr gr lv wr ghflgh zkdw lv phdqw eb wkh zrug "gliihuhqw."