21-110: Problem Solving in Recreational Mathematics

Homework assignment 1

Assigned Wednesday, January 13, 2010. Due Wednesday, January 20, 2010.

Work at least **FOUR** of the following problems. All problems are of equal weight. If you submit solutions for more than four problems, you will get credit for your best four.

You are welcome to work with other students, but the solutions you hand in should be written in your own words. You are not allowed to see the paper another student is going to hand in. If you do collaborate with other students, list their names. If you use other sources, cite them. Give credit where credit is due. See the syllabus for more information about academic integrity.

Hints are encrypted with a *Caesar cipher*, in which each letter is replaced by the letter three places ahead in the alphabet, wrapping around to the beginning if necessary. For example, the letter A is encrypted as D, and Y is encrypted as B. To decrypt the hints, move each letter backward three places.

Problem 1. Find my office. Sign your name on the sheet posted outside my office door.

Hint: Vhh wkh vboodexv.

Problem 2. "How many children do you have, and what are their ages?" asks the census taker.

The mother answers, "I have three children. The product of their ages is 36, and the sum of their ages is the same as my house number."

The census taker looks at the house number, thinks for a moment, and responds, "I'm sorry, but I need more information."

"My oldest child likes chocolate ice cream," says the mother.

"Thank you," replies the census taker. "I have all the information I require."

How old are the children?

Hint: Zkb grhv wkh fhqvxv wdnhu qhhg pruh lqirupdwlrq, dqg lq zkdw zdb lv wkh prwkhu'v odvw vwdwhphqw khosixo?

Problem 3. ("Ins and Outs," *Thinking Mathematically*, page 175.) Take a strip of paper and fold it in half several times in the same fashion as in *Paper Strip* (page 4). Unfold it and observe that some of the creases are IN and some are OUT. For example, three folds produce the sequence

in in out in in out out

What sequence would arise from 10 folds (if that many were possible)?

Hint: Lw pdb eh ehwwhu wr ghvfuleh wkh dqvzhu wkurxjk d sdwwhuq wkdq wr zulwh lw doo rxw hasolflwob. Wkh errn kdv vrph vxjjhvwlrqv li brx duh vwxfn.

Problem 4. (Exercise 1.5 from *Problem Solving Through Recreational Mathematics*, page 27.) Six players—Pietrovich, Cavelli, St. Jacques, Smith, Lord Bottomly, and Fernandez—are competing in a chess tournament over a period of five days. Each player plays each of the others once. Three matches are played simultaneously during each of the five days. The first day, Cavelli beat Pietrovich after 36 moves. The second day, Cavelli was again victorious when St. Jacques failed to complete 40 moves within the required time limit. The third day had the most exciting match of all when St. Jacques declared that he would checkmate Lord Bottomly in 8 moves and succeeded in doing so. On the fourth day, Pietrovich defeated Smith.

Who played against Fernandez on the fifth day?

Hint: Wkh "K" qhaw wr wkh sureohp lq wkh errn lqglfdwhv wkdw wkhuh lv d klqw lq wkh Klqwv dqg Vroxwlrqv vhfwlrq rq sdjh 380.

Problem 5. A sculpture in an art gallery consists of a wooden cube suspended from the ceiling by a thin wire. The wire is attached to the cube at one of its corners, so that the cube hangs at an angle. A fly lands on the top corner of the cube (the one at which the wire is attached) at 12:00 noon. Every five minutes thereafter (at 12:05, 12:10, 12:15, and so on) the fly moves along one of the edges of the cube to reach a neighboring corner. The path of the fly around the cube is random (subject to the restriction that it can only move along edges of the cube). What is the probability that the fly will be at the bottom corner of the cube (the corner nearest the floor) at 12:31?

Hint: Krz pdqb wlphv zloo wkh iob kdyh pryhg eb wkhq? Wub frorulqj wkh fruqhuv ri wkh fxeh vrphkrz.

Problem 6. Three guests check into a hotel and ask what the nightly rate is. The clerk says a room costs 30 dollars a night, so each guest gives the clerk ten dollars, and they head up to the room.

A while later, the clerk realizes he overcharged the guests; the room they are staying in is only 25 dollars a night. So he takes five one-dollar bills from the cash box and hands them to the bellhop with instructions to return the money.

On the way up to the room, the bellhop realizes that five dollars cannot be split evenly among three guests, so he pockets two dollars and returns three dollars to the guests.

Now, each of the guests initially paid ten dollars for the room, but later received a dollar back, so effectively each guest paid nine dollars. In total, then, the room cost the guests 27 dollars. With the two dollars the bellhop kept, this comes to 29 dollars. What happened to the missing dollar?

Hint: Vrphwklqj ilvkb lv jrlqj rq khuh. Lv wkhuh uhdoob d groodu plvvlqj?

Problem 7. (Note: The two parts of this question are not meant to be related to each other, apart from the fact that they are both about arranging coins.)

(a) Place 10 coins in five straight lines so that each line contains exactly four coins.

(b) Can you arrange four coins so that if you choose *any* three of them (i.e., no matter which three of the four you pick), the three coins you chose form the corners of an equilateral triangle?

Hint: Zdwfk rxw iru klgghq dvvxpswlrqv brx pdb eh pdnlqj. Wklqn rxwvlgh wkh era.

Problem 8. (From the article "Number Games and Other Mathematical Recreations" in the 15th edition of the Encyclopædia Britannica.) Three travelers were aboard a train that had just emerged from a tunnel, leaving a smudge of soot on the forehead of each. While they were laughing at each other, and before they could look into a mirror, a neighboring passenger suggested that although no one of the three knew whether he himself was smudged, there was a way of finding out without using a mirror.

He suggested: "Each of the three of you look at the other two; if you see at least one whose forehead is smudged, raise your hand." Each raised his hand at once. "Now," said the neighbor, "as soon as one of you knows for sure whether his own forehead is smudged or not, he should drop his hand, but not before."

After a moment or two, one of the men dropped his hand with a smile of satisfaction, saying: "I know."

How did that man know that his forehead was smudged?

Hint: Lpdjlqh brx duh rqh ri wkh phq; zkdw gr brx vhh? Zkdw gr brx nqrz derxw zkdw wkh rwkhu phq vhh? Wkh sdxvh ehiruh wkh pdq dqvzhuv lv lpsruwdqw. Li brx duh vwxfn, pdnh dq dvvxpswlrq derxw brxu rzq iruhkhdg dqg vhh zkhuh wkdw ohdgv brx.