21-110: Problem Solving in Recreational Mathematics

Challenge questions: Friday, February 5, 2010

Question 1. In the course of adding two numbers, sometimes it is necessary to carry a 1. Explain why it is never necessary to carry a 2.

Question 2. In the course of multiplying two numbers, sometimes it is necessary to carry. Explain why it is never necessary to carry a 9.

Question 3. (Problem 1 from *One Hundred Problems in Elementary Mathematics* by Hugo Steinhaus.) We construct a sequence of numbers as follows: The first number is 2, the next is 3,

$$2 \times 3 = 6,$$

the third number of the sequence is 6,
$$3 \times 6 = 18,$$

the fourth number is 1, and the fifth is 8,

$$6 \times 1 = 6, \qquad 1 \times 8 = 8,$$

the sixth number is 6, then follows 8, etc.

This is the sequence which we obtain:

$$2 \smile 3 \smile 6 \smile 1 \smile 8 \smile 6 \smile 8 \ldots$$

The little arcs under the numbers denote the multiplication carried out, the result of which follows the last digit of the sequence. For example, we ought to multiply now 6 by 8 and write down the numbers of the result, namely 4, 8. There will never be a shortage of numbers for multiplication, since the number of arcs is increased by one with each multiplication and the result will yield at least one and often two digits, so that there always appears at least one new digit.

Prove that numbers 5, 7 and 9 never appear in this sequence.