21-301 Combinatorics
Homework 9
Due: Wednesday, November 15

1. The width $w(P)$ of a finite poset is equal to the size of the maximum anti-chain. The hight $h(P)$ is the size of the largest chain. Prove that $w(P)h(P) \geq |P|$.

2. Several companies send members to a conference; the ith company sends m_i members. During the conference, several workshops are organized simultaneously; the jth workshop can accept at most n_j participants. The organizers want to assign participants to workshops so that no two members of the same company are in the same workshop. (The workshop need not be full.) Each member attends one workshop.

   a) Show how to use a flow network for testing if the constraints may be satisfied.
   b) If there are p companies and q workshops indexed in such a way that $m_1 \geq \cdots \geq m_p$ and $n_1 \leq \cdots \leq n_q$. Show that the constraints can be satisfied if $s(q-t) + \sum_{j=1}^{t} n_j \geq \sum_{i=1}^{s} m_i$ for all $0 \leq s \leq p$, $0 \leq t \leq q$.

3. There are n teams playing some sport. At some point in the season we have the following statistics: $w(i), i \in [n]$ denotes the number of games that team i has one so far. $p(i,j), i \neq j \in [n]$ denotes the number of times team i will play team j in the rest of the season. Set up the question: "can team n still end up with the most wins?" as a network flow problem.