Families with few chains

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Sperner's Theorem tells us the size of the largest family in the Boolean lattice that does not contain a comparable pair of sets. Erdős and Katona asked about the following generalisation: given a number M, amongst families of size M which one contains the fewest comparable pairs? Kleitman solved this problem by showing that to minimise the number of comparable pairs one should take sets as close to the middle layer as possible. His conjecture, that the same conclusion holds if we replace comparable pairs by k-chains for any k, is still wide open. We survey recent results on Kleitman's conjecture.

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