The probability of nonexistence of a subgraph in a moderately sparse random graph

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Abstract

We develop a general procedure that finds recursions for statistics counting isomorphic copies of a graph G_0 in the common random graph models $\mathcal{G}(n,m)$ and $\mathcal{G}(n,p)$. Our results apply when the average degrees of the random graphs are below the threshold at which each edge is included in a copy of G_0. This extends an argument given earlier by Wormald for $G_0 = K_3$ with a more restricted range of average degree. For all strictly balanced subgraphs G_0, our results gives much information on the distribution of the number of copies of G_0 that are not in large "clusters" of copies. The probability that a random graph in $\mathcal{G}(n,p)$ has no copies of G_0 is shown to be given asymptotically by the exponential of a power series in n and p, over a fairly wide range of p. A corresponding result is also given for $\mathcal{G}(n,m)$, which gives an asymptotic formula for the number of graphs with n vertices, m edges and no copies of G_0, for the applicable range of m. An example is given, computing the asymptotic probability that a random graph has no triangles for $p = o(n^{-7/11})$ in $\mathcal{G}(n,p)$ and for $m = o(n^{15/11})$ in $\mathcal{G}(n,m)$, extending results of Wormald.