Estimating r-colorability threshold in a random hypergraph: a simple approach to the second moment method[1]

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The talk deals with estimating the probability threshold for r-colorability of a random hypergraph. Let $H(n,k,p)$ denote the classical binomial model of a random k-uniform hypergraph: every edge of a complete k-uniform hypergraph on n vertices is included into $H(n,k,p)$ independently with probability $p \in (0,1)$.

We study the question of estimating the probability threshold for the r-colorability property of $H(n,k,p)$. It is well known that for fixed $r \geqslant 2$ and $k \geqslant 2$, this threshold appears in a sparse case when the expected number of edges is a linear function of n: $p\binom{n}{k} = cn$ for some fixed $c > 0$.

The following result gives a new lower bound for the r-colorability threshold.

Theorem 1. *Let $k \geqslant 4$, $r \geqslant 2$ be integers and $c > 0$. Then there exist absolute constants $C > 0$ and $d_0 > 0$ such that if $\max(r,k) > d_0$ and*

$$c < r^{k-1}\ln r - \frac{\ln r}{2} - \frac{r-1}{r} - C \cdot \frac{k^2 \ln r}{r^{k/3-1}}, \qquad (1)$$

then

$$\mathsf{Pr}\left(H(n,k,cn/\binom{n}{k}) \text{ is } r\text{-colorable}\right) \to 1 \text{ as } n \to +\infty.$$

Theorem 1 improves the previous result from [1] and provides a bounded gap with the known upper bound. The estimate (1) is only $\frac{r-1}{r} + O\left(\cdot \frac{k^3 \ln r}{r^{k/3-1}}\right)$ less than the upper bound from [1]. For the case $r \gg k$, a slightly better result was recently obtained in [3].

The proof of Theorem 1 is based on the new approach to the second moment method. We also provide some extensions of the used technique.

References

[1] M. Dyer, A. Frieze, C. Greenhill, "On the chromatic number of a random hypergraph", *Journal of Combinatorial Theory, Series B*, **113** (2015), 68–122.

[2] P. Ayre, A. Coja-Oghlan, C. Greenhill, "Hypergraph coloring up to condensation", arXiv:1508.01841

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