Neighbour set distinguishing edge colourings from lists of asymptotically optimal size

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Abstract

Let $G = (V, E)$ be a graph. Consider an edge colouring $c : E \to C$. For a given vertex $v \in V$, by $E(v)$ we denote the set of all edges incident with v in G, while the set of colours associated to these under c is denoted as:

$$S_c(v) = \{c(e) : e \in E(v)\}. \quad (1)$$

The colouring c is called *adjacent vertex distinguishing* if it is proper and $S_c(u) \neq S_c(v)$ for every edge $uv \in E$. It exists if only G contains no isolated edges. The least number of colours in C necessary to provide such a colouring is then denoted by $\chi'_a(G)$ and called the *adjacent vertex distinguishing edge chromatic number* of G. Obviously, $\chi'_a(G) \geq \chi'(G) \geq \Delta$, where Δ is the maximum degree of G, while it was conjectured [3] that $\chi'_a(G) \leq \Delta + 2$ for every connected graph G of order at least three different from the cycle C_5. Hatami [1] proved the postulated upper bound up to an additive constant by showing that $\chi'_a(G) \leq \Delta + 300$ for every graph G with no isolated edges and with maximum degree $\Delta > 10^{20}$.

Suppose now that every edge $e \in E$ is endowed with a list of available colours L_e. The *adjacent vertex distinguishing edge choice number* of a graph G (without isolated edges) is defined as the least k so that for every set of lists of size k associated to the edges of G we are able to choose colours from the respective lists to obtain an adjacent vertex distinguishing edge colouring of G. We denote it by $\mathrm{ch}'_a(G)$. Analogously as above, $\mathrm{ch}'_a(G) \geq \mathrm{ch}'(G)$, while the best (to my knowledge) general result on the classical edge choosability implies that $\mathrm{ch}'(G) = \Delta + O(\Delta^{\frac{1}{2}} \log^4 \Delta)$, see [2]. Extending the thesis of this, a four-stage probabilistic argument granting $\mathrm{ch}'_a(G) = \Delta + O(\Delta^{\frac{1}{2}} \log^4 \Delta)$ for the class of all graphs without isolated edges shall be presented during the talk.

References:

[1] H. Hatami, $\Delta + 300$ is a bound on the adjacent vertex distinguishing edge chromatic number, *J. Combin. Theory Ser. B* **95** (2005) 246–256.

[2] M. Molloy, B. Reed, Near-optimal list colorings, *Random Structures Algorithms* **17** (2000) 376-402.

[3] Z. Zhang, L. Liu, J. Wang, Adjacent strong edge coloring of graphs, *Appl. Math. Lett.* **15** (2002) 623–626.