Spanning universality in random graphs

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Abstract

A graph G is said to be $\mathcal{H}(n,\Delta)$-universal if it contains every graph H on n vertices with maximum degree at most Δ. Using a matching-based embedding technique introduced by Alon and Füredi, Dellamonica, Kohayakawa, Rödl and Ruciński showed that the random graph $G_{n,p}$ is asymptotically almost surely $\mathcal{H}(n,\Delta)$-universal for $p=\tilde{\Omega}(n^{-1/\Delta})$ — a threshold for the property that every subset of Δ vertices has a common neighbour. This bound has become a benchmark in the field and many subsequent results on embedding spanning structures of maximum degree Δ are proven only up to this threshold. We take a step towards overcoming limitations of former techniques by showing that $G_{n,p}$ is almost surely $\mathcal{H}(n,\Delta)$-universal for $p=\tilde{\Omega}(n^{-\varepsilon-1/\Delta})$, for some $\varepsilon=\varepsilon(\Delta)>0$.

This is joint work with Asaf Ferber.