The Logic of Random Perfect Graphs

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A *first order (FO) property* is a graph property that can be expressed by a logic sentence using the quantifiers $\forall, \exists$ with variables ranging over the vertices of the graph, the logical connectives $\wedge, \vee, \neg$, etc., brackets and the relation symbols $=, \sim$, where $x \sim y$ means x and y are connected by an edge. Triangle-freeness is an example of a FO property as it can be written as $\neg \exists x, y, z : (x \sim y) \wedge (x \sim z) \wedge (y \sim z)$.

The study of FO properties of random graphs is a classical, attractive subject going back to the work of Glebskii et al.'69 and independently Fagin'76 who showed that for the Erdös-Rényi random graph $G(n, 1/2)$ we have that

$$\lim_{n \to \infty} \mathbb{P}(G(n, 1/2) \text{ satisfies } \varphi) \in \{0, 1\}, \tag{1}$$

for every FO φ. (By now there is a substantial body of work on FO properties of $G(n, p)$ for other choices of p as well as for other random graph models.)

A perfect graph is a graph for which, in every induced subgraph of it, the chromatic number equals the clique number. Perfect graphs play an important role in modern graph theory, for instance by being the subject of the perfect graph conjecture (solved by Lovász in 1972) and the strong perfect graph theorem (solved by Chudnovsky et al. in 2006).

In this talk we investigate if something similar to (1) might hold for random perfect graphs. That is, we sample a graph G_n uniformly at random from all labelled perfect graphs on n vertices. We show that, perhaps counterintuitively, there exists an FO property φ such that

$$\lim_{n \to \infty} \mathbb{P}(G_n \text{ satisfies } \varphi) \text{ does not exist.}$$

The proof builds on a recent structural description of McDiarmid and Yolov of random perfect graphs and on the seminal work of Shelah and Spencer on FO-properties for the Erdös-Rényi random graph.

(Based on joint work with Marc Noy)