Finitely forcible graph limits are universal

Taísa Martins

Abstract

The theory of graph limits provides analytic tools to analyse large graphs. The theory asserts that large dense graphs can be represented by an analytic object called a graphon; a symmetric measurable function from the unit square to the unit interval. Finitely forcible graphons are graphons that are uniquely determined by finitely many subgraph densities. Such graphons correspond to unique extremal configurations of problems from extremal graph theory.

This close connection with extremal combinatorics has made finitely forcible graphons a subject of much intensive study. In particular, Lovász and Szegedy conjectured that all finitely forcible graphons possess a simple structure in a variety of senses, formalized as [3, Conjectures 9 and 10]:

Conjecture. *The space of typical vertices of every finitely forcible graphon is compact.*

Conjecture. *The space of typical vertices of every finitely forcible graphon has finite dimension.*

Both conjectures were disproved through counterexample constructions [1, 2]. Here, we show that finitely forcible graphons can have arbitrarily complex structure. Our main result reads as follows.

Theorem. *For every graphon W_F, there exists a finitely forcible graphon W such that W_F is a subgraphon of W corresponding to a $1/13$ fraction of the vertices of W.*

Joint work with Jacob Cooper and Dan Král'.

References

[1] R. Glebov, T. Klimošová and D. Král': *Infinite dimensional finitely forcible graphon*, available as arXiv:1404.2743.

[2] R. Glebov, D. Král' and J. Volec: *Compactness and finite forcibility of graphons*, available as arXiv:1309.6695.

[3] L. Lovász and B. Szegedy: *Finitely forcible graphons*, J. Combin. Theory Ser. B **101** (2011), 269–301.