Forbidden vector-valued intersections

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We solve a generalised form of a conjecture of Kalai motivated by attempts to improve the bounds for Borsuk's problem. The conjecture can be roughly understood as asking for an analogue of the Frankl-Rödl forbidden intersection theorem in which set intersections are vector-valued. We discover that the vector world is richer in surprising ways: in particular, Kalai's conjecture is false, but we prove a corrected statement that is essentially best possible, and applies to a considerably more general setting. Our methods include the use of maximum entropy measures, VC-dimension, Dependent Random Choice and a new correlation inequality for product measures.

Joint work with Peter Keevash.