Shattering extremal set systems from Sperner families

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We say that a set system $\mathcal{F} \subseteq 2^{[n]}$ *shatters* a given set $S \subseteq [n]$ if $2^S = \{F \cap S : F \in \mathcal{F}\}$. The Sauer-Shelah lemma states that in general, a set system $\mathcal{F}$ shatters at least $|\mathcal{F}|$ sets. Here we concentrate on the case of equality. A set system is called *shattering-extremal* if it shatters exactly $|\mathcal{F}|$ sets. A conjecture of Rónyai and Mészáros and of Kuzmin and Warmuth states that if a family is shattering-extremal then one can add a set to it and the resulting family is still shattering-extremal. In this talk we discuss how to construct shattering extremal set systems from Sperner families and show how to prove the conjecture for a special class of set systems.