Contagious sets in degree-proportional bootstrap percolation

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We study the following bootstrap percolation process: given a connected graph G on n vertices, we *infect* an initial set $A \subseteq V(G)$, and in each step a vertex v becomes infected if at least a ρ-proportion of its neighbours are infected (where ρ is a fixed constant). Once infected, a vertex remains infected forever. A set A which infects the whole graph is called a *contagious set*. It is natural to ask for the minimal size of a contagious set, which we denote by $h_\rho(G)$. Our main theorem solves the problem by showing that for every $\rho \in (0, 1]$ and every connected graph G of order $n > 1/(2\rho)$ we have $h_\rho(G) < 2\rho n$. This improves the previously best known general upper bound $h_\rho(G) < 4.92\rho n$ and a simple construction shows that this is the best possible bound of this form. We also give a stronger bound for the special case where G has girth at least five, showing that for every $\varepsilon > 0$ and sufficiently small ρ, under this additional assumption we have $h_\rho(G) < (1 + \varepsilon)\rho n$; this bound is asymptotically best-possible.