Disproof of a conjecture of Alon and Spencer

Hüseyin Acan
(Rutgers University)

What is the size of a largest family of edge-disjoint k-cycles that can be packed into the random graph $G = G_{n,1/2}$? Alon and Spencer conjectured the expected value of this number to be $\Omega(n^2/k^2)$ when k is just a little smaller than the clique number of G. We disprove this conjecture by showing that the expected value in question is $O(n^2/k^3)$.

Our main interest lies in answering the following more general question. Let $k \ll \sqrt{n}$ and $A_1, \ldots, A_t$ be random k-subsets of $[n]$, chosen uniformly and independently. Then what can we say about the probability

$$q = q(k, n, t) := \mathbb{P}\big(|A_i \cap A_j| \leq 1 \ \forall i \neq j\big)\,?$$

In particular, is it close to the value obtained by pretending the events $\{|A_i \cap A_j|\}_{i<j}$ are independent? We provide some upper bounds for q and use one of these bounds for the disproof of the packing conjecture. (In that case we only need $k \sim 2\log_2 n$.)

This is joint work with Jeff Kahn.