On the maximum running time in graph bootstrap percolation

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Abstract

Graph bootstrap percolation is a simple cellular automaton introduced by Bollobás in 1968. Given a graph H and a set $G \in E(K_n)$ we initially "infect" all edges in G and then, in consecutive steps, we infect every $e \in K_n$ that completes a new infected copy of H in K_n. We say that G percolates if eventually every edge in K_n is infected. The extremal question about the size of the smallest percolationg sets when $H = K_r$ was answered independently by Alon, Kalai and Frankl. In this paper we investigate further the extremal properties of K_r-bootstrap percolation and we analyse the maximum time the process can run before it stabilizes. It is an easy observation that for $r = 3$ this maximum is $\lceil \log_2(n-1) \rceil$. However, a new phenomenon occurs for $r = 4$ when, as we show, the maximum time of the process is $n-3$. For $r \geq 5$ the behaviour of the dynamics is even more complex which we demonstrate by showing that the K_r-bootstrap process can run for at least $n^{2-\varepsilon_r}$ time steps for some ε_r that tends to 0 as $r \to \infty$.

Joint work with Béla Bollobás, Oliver Riordan and Julian Sahasrabudhe