Packing, Counting and Covering Hamilton cycles in random directed graphs

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Abstract

A Hamilton cycle in a digraph is a cycle passes through all the vertices, where all the arcs are oriented in the same direction. The problem of finding Hamilton cycles in directed graphs is well studied and is known to be hard. One of the main reasons for this, is that there is no general tool for finding Hamilton cycles in directed graphs comparable to the so called Posá 'rotation-extension' technique for the undirected analogue. Let $\mathcal{D}(n,p)$ denote the random digraph on vertex set $[n]$, obtained by adding each directed edge independently with probability p. Here, we present a general and a very simple method, using known results, to attack problems of packing and counting Hamilton cycles in random directed graphs, for every edge-probability $p > \log^C(n)/n$.

Our results are asymptotically optimal with respect to all parameters and apply equally well to the undirected case.

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