EVOLUTION OF HIGH-ORDER CONNECTED COMPONENTS IN RANDOM HYPERGRAPHS

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Given integers $1 \leq j < k$ and a k-uniform hypergraph $\mathcal{H} = (V, E)$ (where $E \subseteq \binom{V}{k}$), we define the notion of j-*connectivity* as follows: Two j-sets J, J' are j-connected if we can walk from J to J' using edges that consecutively intersect in at least j vertices. A j-component is a maximal set of pairwise j-connected j-sets. The case $j = 1$ and $k = 2$ corresponds to connectedness in graphs. More generally, the case $j = 1$ is known as vertex-connectivity.

A priori, studying high-order connected components in (random) hypergraphs is much more involved than analysing their vertex-connected counterparts. However, a powerful tool — the *smoothness lemma* — allows for a very clean treatment of high-order connectivity. This lemma will be discussed in detail by Oliver Cooley in another talk. In the present talk we display its impact on the analysis of the evolution of high-order connected components in k-uniform random hypergraphs.

In [3] the threshold for the existence of a giant component in the k-uniform binomial random hypergraph $\mathcal{H}^k(n, p)$ was determined. In [1] we extend this result by determining the asymptotic size of the giant component shortly after its emergence, i.e. in the very delicate *weakly supercritical* regime. The proof uses a branching process approach.

A second application arises when investigating the k-uniform random hypergraph process $\{\mathcal{H}^k(n, M)\}_M$. In [2] we provide a simple proof of the following hitting time result: With probability tending to one as $n \to \infty$ the hypergraph $\mathcal{H}^k(n, M)$ becomes j-connected exactly at the moment when the last isolated j-set disappears. As a consequence we also establish the threshold for j-connectivity in $\mathcal{H}^k(n, p)$. The corresponding results for vertex-connectivity have previously been established by Poole [4].

The talk is self-contained and therefore also accessible to those who couldn't attend the talk by Oliver Cooley. It is based on joint work with Oliver Cooley and Mihyun Kang.

REFERENCES

1. O. Cooley, M. Kang, and C. Koch, *The size of the giant component in random hypergraphs*, submitted, arXiv:1501.07835.
2. ———, *Threshold and hitting time for high-order connectivity in random hypergraphs*, submitted, arXiv:1502.07289.
3. O. Cooley, M. Kang, and Y. Person, *Giant components in random hypergraphs*, manuscript, arXiv:1412.6366.
4. D. Poole, *On the strength of connectedness of a random hypergraph*, manuscript, arXiv:1409.1489.