Title: Bootstrap percolation on a graph with random and local connections

Abstract:

We consider bootstrap percolation on a graph which is the superposition of a one dimensional lattice and the random graph $G_{n,p}$.

Bootstrap percolation is a process of spread of "activation" on a given realisation of the graph with a given number of initially active nodes. At each step, those vertices which have not been active but have at least 2 active neighbours become active as well.

We consider the n vertices ordered on a ring with deterministic and local connections to the two nearest neighbours, say $i-1$ and $i+1$ for the vertex i, and global connections inherited from the structure of the graph $G_{n,p}$, meaning that any two vertices share an edge with probability p independently of the others.

We study the size A^* of the final active set depending on the number of vertices active at the origin as a function of n and p which we write $a_0(n,p) = a_0$. The process exhibits a phase transition. Moreover, we show that the process can percolate $A^* = n - o(n)$ while the process restricted to $G_{n,p}$ stays subcritical $A^*_{|_{G_{n,p}}} \leq 2a_0$.

We will review in a first time the results derive on $G_{n,p}$ and compare the critical parameters with the one for the superposition model.