Rapid mixing for the non-critical random-cluster model on the square lattice

Mario Ullrich Friedrich Schiller University Jena email: mario.ullrich@uni-jena.de

Abstract

We consider the random-cluster model with parameters $p \in (0, 1)$ and $q \in \mathbb{N}$ on finite boxes in the two-dimensional square lattice for non-critical values of p, that is $p \neq p_c = \sqrt{q}/(1 + \sqrt{q})$. We prove that the spectral gap of the continuous-time heat-bath dynamics for this model with free or wired boundary conditions is bounded from below by a constant.

The proof uses the standard technique of block dynamics, which is also a main tool for proving the analogous results for the corresponding Potts (or Ising) model. But some crucial assumptions for this analysis are not valid in our case, since, unlike the Potts model, the random-cluster model does not have finite range of interactions and strong (spatial) mixing in the desired range of p.

Finally, we will discuss the (possible) limitations of the approach if we consider $q \notin \mathbb{N}$ or arbitrary boundary conditions.