Inapproximability for Antiferromagnetic Spin Systems in the Tree Non-Uniqueness Region

Andreas Galanis* Daniel Štefankovič†(speaker) Eric Vigoda*

Abstract

A remarkable connection has been established for 2-spin systems, including the Ising and hard-core models, showing that the computational complexity of approximating the partition function for graphs with maximum degree Δ undergoes a phase transition that coincides with the statistical physics uniqueness/non-uniqueness phase transition on the infinite Δ-regular tree. Despite this clear picture for 2-spin systems, there is little known for multi-spin systems. We present an analog of the above inapproximability results for multi-spin systems. We prove that, unless $NP = RP$, for any antiferromagnetic spin system, there is no FPRAS for the partition function of Δ-regular graphs when the dominant semi-translation invariant Gibbs measures on the infinite Δ-regular tree are not translation invariant and are permutation symmetric of each other. Our results apply to the antiferromagnetic Potts model (even q) and colorings problem (even k), which are the multi-spin systems of particular interest. Our proof relies on a simple and generic analysis of the second moment for any spin system. As a consequence we get concentration results for any spin system in which one can analyze the first moment. We also present a tool for simplifying the associated first moment calculations by relating it to the stable fixed points for the tree recursions.

*School of Computer Science, Georgia Institute of Technology, Atlanta GA 30332.
†Department of Computer Science, University of Rochester, Rochester, NY 14627.