Randomized Approximation of b-Matching in Hypergraphs

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Abstract Let (V, E) be a hypergraph where V is a finite set (set of nodes) and $E \subseteq$ is collection of subsets of V (set of hyperedges) with $|V| = n$ and $|E| = m$. Let $b \in \mathbb{N}_{\geqslant 1}$. We call a set $M \subseteq E$ a *b-matching* if no node is contained in more than b edges from M. Maximum b-Matching is the problem of finding a b-matching with maximum cardinality, which we denote by OPT_b. b-Matching is a classical problem in combinatorics and optimization, and in graphs it has been a driving area in combinatorial optimization. In hypergraphs the problem is NP-hard and tight approximation algorithms are sought. With variants of the *randomized rounding scheme* of Raghavan and Thompson (1987) several randomized and derandomized constant-factor approximations, that is the construction of a matching of cardinality at least $c \cdot \mathrm{OPT}_b$, where $c < 1$ is a constant, have been presented (e.g. Srivastav, Stangier 1996 and Srinivasan 1999), provided that b is large, namely $b \geqslant \alpha \log n$, and $\alpha > 0$ constant. The question is whether approximations are possible for small b's. With the FKG correlation inequality approximations of the type $\Omega(\alpha(n, b)\,\mathrm{OPT}_b)$ were first proved by Srinivasan (1999) (for recent improvements see Bansal, Korula, Nagarajan, Srinivasan 2012), where the approximation factor is of the form $\alpha(n, b) = (\mathrm{OPT}_b/n)^{1/b}$. Note that $\alpha(n, b)$ is constant for hypergraphs with $\mathrm{OPT}_b = \Theta(n)$ for any b. But otherwise if $\mathrm{OPT}_b << n$ and $b << \log n$, it is only $o(1)$ for all instances, thus is negligible.

In this paper we show that a randomized worst-case approximation with a constant-factor *independent of the instance* is possible for any $b \geqslant \alpha\sqrt{\log n}$, $\alpha > 0$ some constant. The analysis of the randomized algorithm depends on the martingale inequality of Azuma. Furthermore, we present new algorithms of *hybrid* type, where the randomized rounding is further improved by greedy heuristics. In an experimental study within the framework of *Algorithm Engineering* we show that such hybrid algorithms outperform all known approximation algorithms.

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