Equitable two-colorings of uniform hypergraphs

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An equitable two-coloring of a hypergraph $H = (V, E)$ is a proper vertex two-coloring such that the cardinalities of color classes differ by at most one. In connection with property B problem Radhakrishnan and Srinivasan proved that if H is a k-uniform hypergraph with maximum vertex degree $\Delta(H)$ satisfying

$$\Delta(H) \leqslant c\,\frac{2^{k-1}}{\sqrt{k\,\ln k}} \tag{1}$$

for some absolute constant $c > 0$, then H is 2-colorable. By using the Lovász Local Lemma for negatively correlated events and the random recoloring method we prove that if H either is a simple hypergraph (every two distinct edges have at most one common vertex) or has a lot of vertices, then under the same condition (1) on the maximum vertex degree it has an equitable coloring with two colors.