Coupling, random intersection graphs and Hamilton cycles.

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In a random intersection graph $\mathcal{G}(n,m,p)$ defined by Karoński, Scheinerman and Singer-Cohen there is a set of n vertices $\mathcal{V}$ and an auxiliary set of $m = m(n)$ features $\mathcal{W}$. Each vertex $v \in \mathcal{V}$ adds a feature $w \in \mathcal{W}$ to its feature set $W(v)$ with probability p independently of all other vertices and features. Any two vertices $v, v' \in \mathcal{V}$ are connected by an edge in $\mathcal{G}(n,m,p)$ if $W(v)$ and $W(v')$ intersect. $\mathcal{G}(n,m,p)$ may be generalised by taking different probabilities p for distinct features.

We establish a sharp threshold function for Hamilton cycles for $\mathcal{G}(n,m,p)$. In the proof we use a coupling which establish relations between generalised $\mathcal{G}(n,m,\overline{p})$ and an auxiliary random graph similar to Erdős and Rényi random graph. Moreover, in order to check the minimum degree condition, we use a coupling of the construction of $\mathcal{G}(n,m,\overline{p})$ with the coupon collector process. This is one of many applications of the above stated couplings. Presented technique refine considerably previous one, which used coupling with Erdős and Rényi random graph.