Contagious sets in expanders

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Abstract

We consider the following activation process in d-regular undirected graphs: a vertex is active either if it belongs to a set of initially activated vertices or if at some point it has at least r active neighbors, where $1 < r \leq d$ is the activation threshold. Such processes have been studied extensively in several fields such as combinatorics, computer science, probability and statistical physics. A *contagious set* is a set whose activation results with the entire graph being active. Given a graph G, let $m(G, r)$ be the minimal size of a contagious set.

We present upper bounds on $m(G, r)$ on d-regular graphs with expansion properties (parameterized by the spectral gap and/or the girth of the graphs). In some cases we also provide nearly matching lower bounds. The general flavor of our results is that sufficiently strong expansion (i.e. $\lambda(G) = O(\sqrt{d})$) or sufficiently large girth (that is, girth $\Omega(\log \log d)$) implies that in n-vertex graphs, $m(G, 2) \leq O(\frac{n}{d^2})$. Furthermore, we show that in the absence of 4-cycles, $\lambda(G) < (1 - \epsilon)d$ ensures that $m(G, 2) = O(\frac{n \log d}{\epsilon^2 d^2})$. Time permitting, we shall discuss several open problems arising from our work.

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