On new family of expander graphs and their applications

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Expander graphs are highly connected, sparse, finite graphs. The property of being an expander seems significant in many of these mathematical, computational and physical contexts. Even more, expanders are surprisingly applicably applicable in other computational aspects: in the theory of error correcting codes and the theory of pseudorandomness, which are used in probabilistic algorithms. Constructions of the best expander graphs with a given regularity and order is not easy and in many cases, it is an open problem. In this presentation we present a method to obtain a new examples of families of expanders graphs and some examples of Ramanujan graphs which are the best expanders. We describe properties of obtained graphs in comparison to previously known results. Numerical computations of eigenvalues presented in this abstract have been computed with MATLAB.

By using algebra we can construct arbitrary large (for arbitrary n), sparse, bipartite, $q+1$-regular graphs with $|V| = 2(1+q+q^2+\ldots+q^n)$ vertices and $(q+1)(1+q+q^2+\ldots+q^n)$ edges. Many of the obtained constructions are expanders graphs and some of them are Ramanujan graphs.

Each of the representatives of the presented family is $q+1$-regular graph so the first eigenvalue of the adjacency matrix, corresponding to this graph, is $\lambda_0 = q+1$. Each of the representatives of the presented family is $q+1$-regular graph so the first eigenvalue of the adjacency matrix, corresponding to this graph, is $\lambda_0 = q+1$. Let us denote the second eigenvalue by $\lambda_1 = \max_{\lambda_i \neq q+1} |\lambda_i|$. Table below shows expanding properties of some representatives of constructed graphs.

Number field	λ_0	λ_1	$2\sqrt{q}$	$\|V\|$
$\mathbb{F}_2$	3	2.4495	2.8284	62
$\mathbb{F}_3$	4	3.2004	3.4641	242
$\mathbb{F}_4$	5	3.9180	4	682
$\mathbb{F}_5$	6	4.1317	4.4721	1562
$\mathbb{F}_7$	8	4.8887	5.2915	5602
$\mathbb{F}_{11}$	12	6.1283	6.6332	32210

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