Fixed-point polynomials of permutation groups

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joint work with Christopher Harden

Given a (finite) group G of permutations of a finite set Ω with $|\Omega| = n$, we can let f_i be the number of elements of the group which fix exactly i elements of Ω, and use these to form the polynomial $\sum_{i=0}^{n} f_i x^i$. This polynomial is also (a constant times) the probability generating function of the number of fixed points of a random element of the group.

In this work, at least partly motivated by the recent work on how the location of roots of chromatic polynomials gives information about the underlying graphs, we investigate links between the roots of these polynomials and the structure of the permutation group. In particular, I will make some remarks about the polynomials arising when a group acts on itself by conjugation. There are a number of curious 'random-like' properties which the polynomials often have.

Reference C. M. Harden and D. B. Penman. Fixed Point Polynomials of Permutation Groups. *Electron J. Comb.* **20** (2013) issue 2, P26