POLYNOMIAL-TIME PERFECT MATCHINGS IN DENSE HYPERGRAPHS

RICHARD MYCROFT

A well-known theorem of Rödl, Ruciński and Szemerédi states that any k-graph H on n vertices with minimum codegree $\delta(H) \geq n/2 + C$ contains a perfect matching. Indeed, for large n the theorem establishes precisely the best-possible value of C for which this statement holds. We therefore cannot be sure that a k-graph H satisfying a weaker minimum degree condition will contain a perfect matching. However, we might hope to prove that such a k-graph either contains a perfect matching or has a given extremal structure for which there is no perfect matching. That is, we would like to characterise those k-graphs which satisfy some weaker minimum degree condition but have no perfect matching.

In this talk I will present such a characterisation for k-graphs H with $\delta(H) \geq n/k + o(n)$. For $k = 3$ this characterisation has a simple formulation: for any 3-graph H with $\delta(H) \geq n/3 + o(n)$, either H contains a perfect matching or there exists some $A \subseteq V(H)$ such that $|A|$ is odd but $|e \cap A|$ is even for any $e \in E(H)$. Unfortunately, the naive generalisation of this result for $k \geq 4$ is false; our characterisation for these values of k is more complicated.

I will also outline a polynomial-time algorithm which tests for this characterisation. As a consequence, we can determine in polynomial time whether or not a k-graph H on n vertices with $\delta(H) \geq n/k + o(n)$ contains a perfect matching. Furthermore, by derandomising a relatively straightforward random algorithm, we can repeatedly use this testing algorithm to find a perfect matching in such an H in polynomial time (if one exists).

Let $\mathrm{PM}(k, \delta)$ denote the decision problem of determining whether or not a k-graph H on n vertices with $\delta(H) \geq \delta n$ contains a perfect matching. So the results described above imply that $\mathrm{PM}(k, \delta)$ is in P for any $\delta > 1/k$. This essentially answers a problem of Karpiński, Ruciński and Szymańska, who had previously shown the existence of ε such that $\mathrm{PM}(k, 1/2-\varepsilon)$ is in P. Indeed, Szymańska gave an elegant reduction proving that $\mathrm{PM}(k, \delta)$ is NP-complete for any $\delta < 1/k$, so the minimum codegree threshold at which the perfect matching problem becomes tractable is asymptotically n/k.

This is joint work with Peter Keevash and Fiachra Knox. (For simplicity, the condition $k \mid n$ has been omitted throughout this abstract; this is a necessary condition for a k-graph to contain a perfect matching.)

School of Mathematics, University of Birmingham, Birmingham B15 2TT, United Kingdom