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Small minors and topological minors

For every integer t there is a smallest real number $c(t)$ such that any graph with n vertices and more than $c(t)n$ edges must contain a K_t-minor. Fiorini, Joret, Theis and Wood conjectured that if a graph on n vertices has more than $(c(t)+\varepsilon)n$ edges then it must contain a K_t-minor formed from at most $C(\varepsilon)\log n$ vertices. Shapira and Sudakov subsequently proved that such a graph contains a K_t-minor formed from at most $C(\varepsilon)\log n \log\log n$ vertices. Building on their method, we will remove the $\log\log n$ factor and prove the conjecture. Graphs of constant degree yet high girth demonstrate that this result is tight up to the constant factor. We will also outline a similar result with topological minors or subdivisions.

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