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AN ANALYTIC COMPARATIVE METHOD FOR DECOMPOSABLE STRUCTURES

The celebrated Flajolet-Odlyzko method applied in the analytic theory of decomposable structures requires rather stringent regularity properties of their generating functions. This is becoming a serious obstacle even dealing with random permutations taken according to a generalized Ewens probability. The latter is motivated by several recent applications in the Bose gas theory. Since 2002 we [1] are developing comparative asymptotic analysis of two power series coefficients. The approach works under some averaged conditions posed on parameters of classes of decomposable structures such as assemblies, weighted multisets, or additive arithmetical semigroups.

To illustrate this, let us explore the symmetric group $\mathbb{S}_n$ of order $n \in \mathbb{N}$. Denote by $k_j(\sigma) \in \mathbb{N}_0$ the number of cycles of length $1 \le j \le n$ in the permutation $\sigma \in \mathbb{S}_n$. Assume that σ is taken at random with a probability

$$\nu_n^{(\theta)}(\{\sigma\}) := \Lambda_n^{-1} \prod_{j=1}^{n} \theta_j^{k_j(\sigma)}, \quad \sigma \in \mathbb{S}_n,\ 0^0 := 1,$$

where $\theta_j := \theta_j(n) \ge 0$ if $j \le n$ and Λ_n is an appropriate normalizing sequence. To develop a comprehensive theory containing the value distribution analysis for functions defined on such permutations, it suffices to assume averaged conditions involving θ_j and D_n defined by

$$\exp\Big\{ \sum_{j \le n} \frac{\theta_j}{j} z^j \Big\} =: \sum_{k=0}^{\infty} D_k z^k.$$

The recent results obtained in [2] and in the references therein will be surveyed in the talk.

References

[1] E. Manstavičius, Mappings on decomposable combinatorial structures: analytic approach, *Combinatorics, Probab. Computing*, **11**, 61–78 (2002).

[2] E. Manstavičius, On total variation distance for random assemblies, *Discrete Math. and Theor. Computer Sci.*, AofA'12 Proc., 97–108 (2012).