Decreasing the Chromatic Number of a Random Graph

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Abstract

We consider the chromatic number of the binomial random graph $G_{n,p}$ and observe a strengthening of the classical result that $\chi(G_{n,p}) = (1+o(1))n/2\log_b(np)$ a.a.s. where $b = 1/(1-p)$. For p bounded away from 1 and $np \geq d_0$ for some d_0 large enough, the property

$$\chi(G_{n,p}) \geq (1+o(1))\frac{n}{2\log_b(np)} \text{ a.a.s.}$$

is resilient to the removal of any subgraph H with maximum degree $o(\log(np))$. The same assertion holds with H a subgraph having $o(n\log(np))$ edges. Both assertions are best possible, up to a constant factor. These statements follow from a connection between the chromatic number's (removal) resilience and improper chromatic graph parameters.

This provides a counterpoint to work of Alon and Sudakov (2010) analysing the resilience of $\chi(G_{n,p})$ subject to the *addition* of edges.

This is based on joint work with Colin McDiarmid.