Discontinuous bootstrap percolation in power-law random graphs

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Bootstrap percolation processes were introduced in the late 1970s by physicists in the context of magnetic disordered systems. It is a type of infection process in graphs which is determined by the following simple rule: if an uninfected vertex has at least r infected neighbours, then it becomes infected and remains so forever. Here, r is a fixed positive integer, which does not depend on the number of vertices of the underlying graph and the initial set of infected vertices is random. We will discuss the behaviour of this class of processes on random graphs that are inhomogeneous, that is, the probability that a certain pair of vertices appears as an edge of the graph depends on the vertices. The special class of these random graphs are the so-called Chung-Lu random graphs which exhibit power-law degree distribution. We establish a phase transition on the evolution of the process when the number of initially infected vertices "crosses" a certain function, which we determine. Namely, we show that when the exponent of the power law is between 2 and 3 a sublinear number of initially infected vertices may spread the infection to the a linear number of vertices. We establish a law of large numbers for the size of the infected set at the end of the process.

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