The minimum number of disjoint pairs in set systems and related problems

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Let $\mathcal{F}$ be a family of subsets of $[n]$ such that all sets have size k and every pair of sets intersect. The celebrated theorem of Erdős-Ko-Rado from 1961 says that when $n \geq 2k$, any such family has size at most $\binom{n-1}{k-1}$. A natural question to ask is how many disjoint pairs must appear in a set system of larger size. In 1978, Ahlswede and Katona resolved this question for $k = 2$.

In this talk, we shall determine the minimum number of disjoint pairs in small k-uniform families, thus confirming a conjecture of Bollobás and Leader. Moreover, we obtain similar results for two well-known extensions of the Erdős-Ko-Rado theorem, determining the minimum number of matchings of size q and the minimum number of t-disjoint pairs that appear in set systems larger than the corresponding extremal bounds. In the latter case, this provides a partial solution to the Kleitman-West problem.

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