On the component structure of random geometric graphs on the hyperbolic plane

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This work is a study of a family of random geometric graphs on the hyperbolic plane. Here, N points are chosen randomly on a disc of radius R on the hyperbolic plane and any two of them are joined by an edge if they are within hyperbolic distance R. The parameter R is suitably chosen so that the disc in which the N points are chosen has volume that is proportional to N^2. The distribution of the N points is *quasi-uniform*, which is a distorted version of the uniform distribution and is controlled by a parameter α – when $\alpha = 1$ this coincides with the uniform distribution. Krioukov et al. [2] recently introduced this as a model for complex networks as it displays basic properties of these such as a power law degree distribution and clustering as an expression of the underlying hyperbolic geometry.

The present paper focuses on the evolution of the component structure of the random graph as this is determined by α. In particular, we focus on two phase transitions with sharp thresholds at $\alpha = 1$ and at $\alpha = 1/2$. For $\alpha > 1$, we show that with high probability as N grows the largest component of the random graph has sublinear order. When α crosses 1 a "giant" component of linear order emerges. But if $\frac{1}{2} < \alpha < 1$, then the random graph is still disconnected with high probability. We show that the value $1/2$ is the connectivity threshold: when $\alpha < \frac{1}{2}$, the random graph is connected with high probability.

It has been shown by Gugelmann et al. [1] that when $\alpha > 1$ the random graph has power law degree distribution with exponent larger than 3. Our results imply that in this case, the random graph has no giant component independently of its average degree. This is a behaviour that is in sharp contrast with the classical Erdős-Rényi model as well as with random geometric graphs on Euclidean spaces.

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References

[1] L. Gugelmann, K. Panagiotou and U. Peter, Random hyperbolic graphs: degree sequence and clustering, In *Proceedings of the 39th International Colloquium on Automata, Languages and Programming (A. Czumaj et al. Eds.)*, Lecture Notes in Computer Science 7392, 2012, pp. 573–585.

[2] D. Krioukov, F. Papadopoulos, M. Kitsak, A. Vahdat and M. Boguñá, Hyperbolic Geometry of Complex Networks, *Phys. Rev. E* **82** (2010), 036106.