Achlioptas Process Phase Transitions Are Continuous

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Abstract

It is widely believed that certain simple modifications of the random graph process lead to discontinuous phase transitions. In particular, starting with the empty graph on n vertices, suppose that at each step two pairs of vertices are chosen uniformly at random, but only one pair is joined, namely one minimizing the product of the sizes of the components to be joined. Making explicit an earlier belief of Achlioptas and others, in 2009, Achlioptas, D'Souza and Spencer conjectured that there exists a $\delta > 0$ (in fact, $\delta \geq 1/2$) such that with high probability the order of the largest component 'jumps' from $o(n)$ to at least δn in $o(n)$ steps of the process, a phenomenon known as 'explosive percolation'.

We give a simple proof that this is not the case. Our result applies to all 'Achlioptas processes', and more generally to any process where a fixed number of independent random vertices are chosen at each step, and (at least) one edge between these vertices is added to the current graph, according to any (online) rule.

We also prove the existence and continuity of the limit of the rescaled size of the giant component in a class of such processes, settling a number of conjectures. Intriguing questions remain, however, especially for the product rule described above.

Joint work with Oliver Riordan.