Sharp Mixing Time Bounds for Sampling Random Surfaces

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Abstract

We analyze the mixing time of a natural local Markov Chain (Gibbs sampler) for two commonly studied models of random surfaces: (i) discrete monotone surfaces in $\mathbb{Z}^3$ with almost planar boundary conditions and (ii) the one-dimensional discrete Solid-on-Solid (SOS) model.

In both cases we prove the first almost optimal bounds

$$O(L^2 \text{polylog}(L))$$

where L is the natural size of the system. Our proof is inspired by the so-called mean curvature heuristic: on a large scale, the dynamics should approximate a deterministic motion in which each point of the surface moves according to a drift proportional to the local inverse mean curvature radius. Key technical ingredients are monotonicity, coupling and an argument due to D.Wilson in the framework of lozenge tiling Markov Chains together with Kenyon's results on the free Gaussian field approximation of monotone surfaces.

The novelty of our approach with respect to previous results consists in proving that, with high probability, the dynamics is dominated by a deterministic evolution which, apart from polylog(L) corrections, follows the mean curvature prescription. Our method works equally well for both models despite the fact that their equilibrium maximal deviations from the average height profile occur on very different scales ($\log(L)$) for monotone surfaces and $L^{1/2}$ for the SOS model).