Tree Universality in Random Graphs

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Abstract

A classical result by Erdős and Rényi states that asymptotically almost surely (a.a.s.) a random graph G in the $G(n,p)$ model is connected if the expected degree pn of its vertices is slightly larger than $\log n$. Naturally, this also implies that such a graph contains a spanning tree.

Recently, Krivelevich showed that if T is a n-vertex tree with bounded maximum degree, then G a.a.s. contains T if pn is at least of order n^ϵ where ϵ is an arbitrary positive constant. However, this does not imply that G a.a.s. contains all such trees simultaneously, i.e., that G is universal for all n-vertex trees with bounded maximum degree. We study the question for which values of pn such a universality occurs.

To this end, we investigate sparse n-vertex graphs that satisfy certain natural expansion properties. We show that a graph with these properties does indeed contain every n-vertex tree with bounded maximum degree. We then see that a random graph drawn according to the $G(n,p)$ model with pn at least of order $n^{2/3}\log^2 n$ a.a.s. has the desired expansion properties and is therefore universal for the class of n-vertex trees with bounded maximum degree.

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