Boolean functions in the implicational fragment:
On the Number of Boolean Expressions and Its Relation to the Function Complexity

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Abstract

We consider the logical system of Boolean expressions built on the single connector of implication and on positive literals. Assuming all expressions of a given size to be equally likely, we prove that we can define a natural probability distribution on the set of Boolean functions expressible in this system. Then we show how to approximate the probability of a function f when the number of variables grows to infinity, and that this asymptotic probability has a simple expression in terms of the complexity of f. We also prove that most expressions computing any given function in this system are simple in a certain sense.

Based on joint work with Hervé Fournier, Danièle Gardy, and Antoine Genitrini.