Average-case Performance of Heuristics for Three-dimensional Random Assignment Problems

Alan Frieze, Carnegie Mellon University

Abstract

Beautiful formulas are known for the expected cost of random two-dimensional assignment problems, but in higher dimensions, even the scaling is not known. In 3 dimensions and above, the problem has natural "planar" and "axial" versions, both of which are NP-hard. For 3-dimensional Planar random assignment instances of size n, the cost scales as $\Omega(1/n)$, and a main result of the present paper is the first polynomial-time algorithm that, with high probability, finds a solution of cost $O(n^{-1+\epsilon})$, for arbitrary positive ϵ (or indeed ϵ going slowly to 0). For 3-dimensional Axial assignment, the lower bound is $\Omega(n)$, and we give a new efficient matching-based algorithm that returns a solution with expected cost $O(n \log n)$.

Joint work with Gregory Sorkin.