On the resilience of Hamiltonicity and optimal packing of Hamilton cycles in random graphs

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Abstract

Let $\mathbf{k} = (k_1, \ldots, k_n)$ be a sequence of n integers. For an increasing monotone graph property $\mathcal{P}$ we say that a base graph $G = ([n], E)$ is $\mathbf{k}$-*resilient* with respect to $\mathcal{P}$ if for every subgraph $H \subseteq G$ such that $d_H(i) \leq k_i$ for every $1 \leq i \leq n$ the graph $G - H$ possesses $\mathcal{P}$. This notion naturally extends the idea of the *local resilience* of graphs recently initiated by Sudakov and Vu. In this paper we study the $\mathbf{k}$-resilience of a typical graph from $\mathcal{G}(n, p)$ with respect to the Hamiltonicity property where we let p range over all values for which the base graph is expected to be Hamiltonian. Considering this generalized approach to the notion of resilience our main result implies several corollaries which improve on the best known bounds of Hamiltonicity related questions. For one, it implies that for every positive $\varepsilon > 0$ and large enough values of K, if $p > \frac{K \ln n}{n}$ then with high probability the local resilience of $\mathcal{G}(n, p)$ with respect to being Hamiltonian is at least $(1 - \varepsilon)np/3$, improving on the previous bound for this range of p. Another implication is a result on optimal packing of edge disjoint Hamilton cycles in a random graph. We prove that if $p \leq \frac{1.02 \ln n}{n}$ then with high probability a graph G sampled from $\mathcal{G}(n, p)$ contains $\lfloor \frac{\delta(G)}{2} \rfloor$ edge disjoint Hamilton cycles, extending the previous range of p for which this was known to hold.