

# Random low degree polynomials are hard to approximate

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## Abstract

We study the problem of how well a typical multivariate polynomial can be approximated by lower degree polynomials over  $Z_2$ . We prove that, with very high probability, a random degree  $d + 1$  polynomial has only an exponentially small correlation with all polynomials of degree  $d$ , for all degrees  $d$  up to  $\Theta(n)$ . That is, a random degree  $d + 1$  polynomial does not admit a good approximation of lower degree. In order to prove this, we provide far tail estimates on the distribution of the bias of a random low degree polynomial. Recently, several results regarding the weight distribution of Reed–Muller codes were obtained. Our results can be interpreted as a new large deviation bound on the weight distribution of Reed–Muller codes.