Magma Dynamics: Media with Large Deformation and Evolving Microstructure

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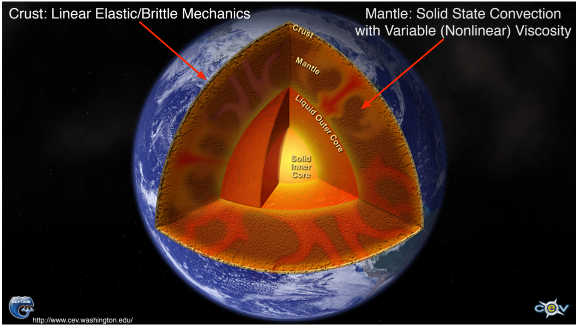
Outline

1. Physical Motivation
2. Models
   - Historical Development–Multiphase Flow
   - Multiscale Development–Homogenization
3. Evolution & Generalizations
   - Evolving Microstructures
   - Sub Grain Scale Processes

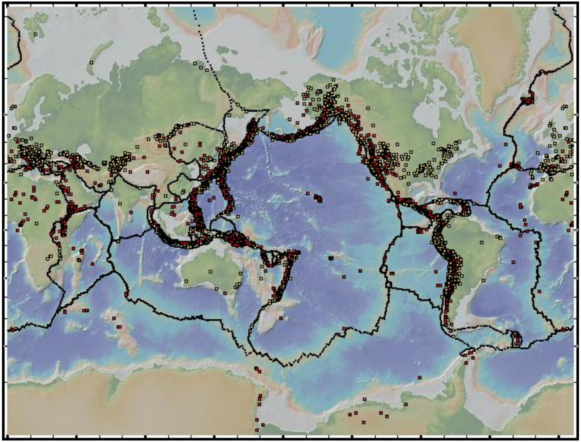
Large Scale Dynamics



Grand Challenge

Explain the formation and motion of the tectonic plates

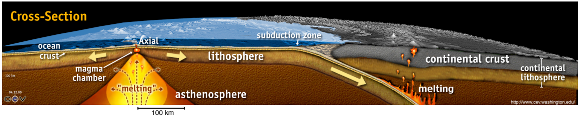
Plate Boundaries & Molten Rock

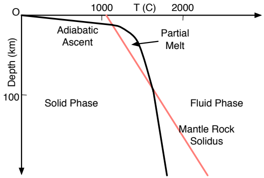


Correlation

Volcanos, seismic events, and the plate boundaries

Magmatic Events & Melt Generation





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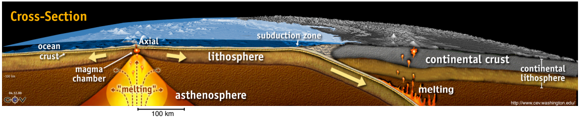

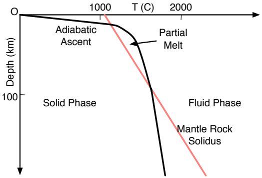

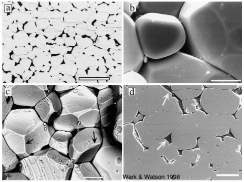

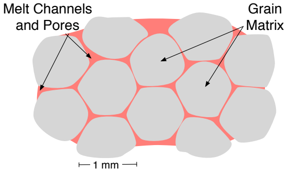

Melt Transport

- Constituent minerals melt at the grain scale,

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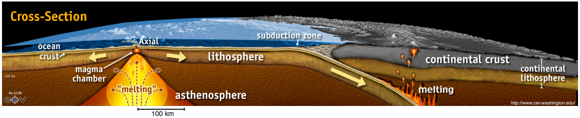

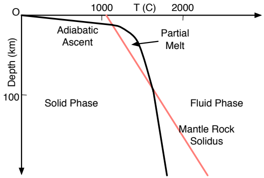

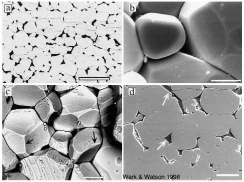

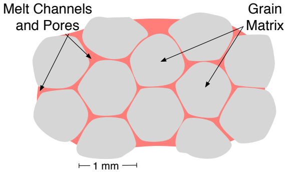

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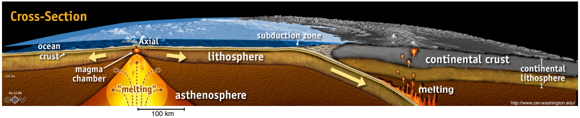
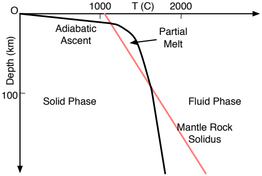
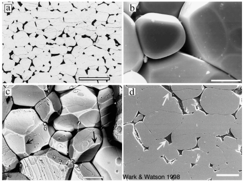
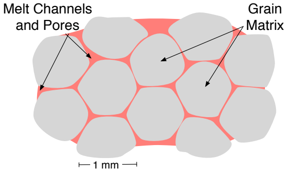






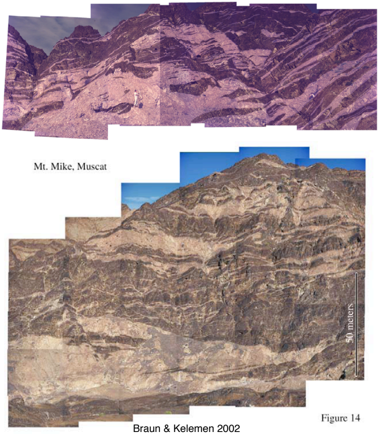


Melt Transport

- Constituent minerals melt at the grain scale,
- Melt focuses and travels to the surface–porous flow,
- Evolving microstructure (viscous compaction/dilation & shearing, advection)

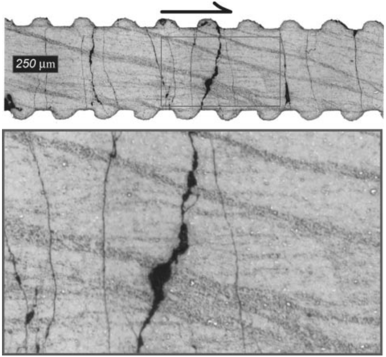
Field & Lab Observations of Deformable Porous Flow

Channelization



Braun & Kelemen 2002

Shear Banding



Holtzman, Kohlstedt, & Morgan 2005

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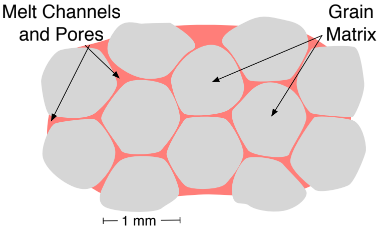
Assumptions



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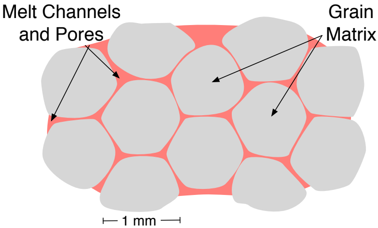

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- Zero Reynolds number; no intertial forces,

$$\cancel{\mathrm{Re.}\left(\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u}\right)} = -\mathrm{Re.}\nabla p + \nabla^2\mathbf{u}, \quad \nabla\cdot\mathbf{u} = 0$$

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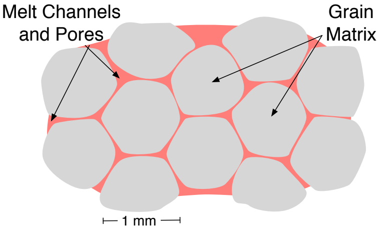

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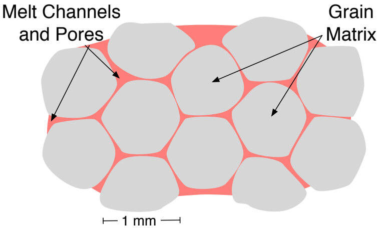

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- Macroscopically, grain matrix is compressible,
- Macroscopic constitutive relations must be assumed

Previous Work

Early Multiphase Flow Models

- McKenzie 1984
- Scott–Stevenson 1984, 1986

Thermodynamic Models

- Fowler 1984, 1985, 1989

McKenzie Refined

- Spiegelman 1993
- Katz *et. al.* 2007

Systematic Multiphase Flow Models+New Physics

- Bercovici–Ricard–Schubert 2001,
- Bercovici–Ricard 2003, 2006, 2007,
- Hier-Majumder–Ricard–Bercovici 2006,
- Takei–Hier-Majumder 2009

Viscously Deformable Porous Flow Equations

Multiphase Flow ("Empirical")

Conservation of Mass

$$\partial_t (\rho_f \phi) + \nabla \cdot (\rho_f \phi \mathbf{V}^f) = \text{melting/freezing} \quad (1)$$

$$\partial_t [\rho_s (1-\phi)] + \nabla \cdot [\rho_s (1-\phi) \mathbf{V}^s] = -\text{melting/freezing} \quad (2)$$

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$$\phi(\mathbf{V}^f-\mathbf{V}^s)=-\frac{K}{\mu_f}(\nabla P-\mathbf{g}^f) \quad (3)$$

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Matrix Stress Balance

$$0 = \overline{\rho}\mathbf{g} - \nabla P + \nabla \cdot \left[(1-\phi)\mu_s \left(\nabla \mathbf{V}^s + (\nabla \mathbf{V}^s)^T\right)\right] + \nabla \left[(1-\phi)(\zeta_s - \tfrac{2}{3}\mu_s)\nabla \cdot \mathbf{V}^s\right] \quad (4)$$

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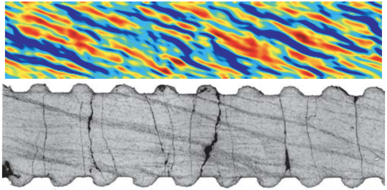
Closures

Permeability, $K \sim \phi^n$, Bulk viscosity, $\zeta_s \sim \phi^{-m}$, other physics

Successes & Challenges

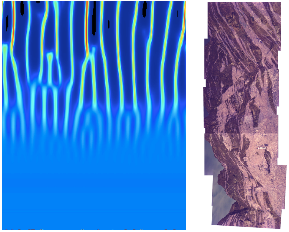
Computational Successes

Katz–Spiegelman–Holtzman 2006



Nonlinear Viscosity

Spiegelman–Kelemen–Aharonov 2001



Reactive Flow

Modeling Challenge

Upscale a grain scale model to a macroscopic one with self-consistent closures.

Results

S.–Spiegelman–Weinstein, JGR–Solid Earth, 2010

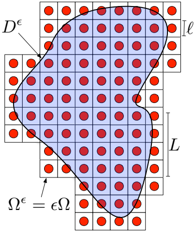
- Macroscopic equations can be derived from a grain scale model of two viscous fluids via homogenization.
- Self-consistent closures for assumed microstructures can be found numerically:

$$\begin{aligned} \text{Permeability: } & k_{\text{eff.}} \propto \phi^{n}, n \sim 2 \\ \text{Bulk Viscosity: } & \zeta_{\text{eff.}} \propto \mu_s \phi^{-m}, m \sim 1 \\ \text{Anisotropic Viscosity: } & \eta_{\text{eff.}} \sim \mathcal{O}(\phi) \end{aligned}$$

- Bulk viscosity is relatively insensitive to microstructure

Homogenization

Bensoussan–Lions–Papanicolaou, Sanchez-Palencia, Auriault,...



- Need PDE valid at the fine scale,

$$L^\epsilon \mathbf{u}^\epsilon + N^\epsilon(\mathbf{u}^\epsilon) = \mathbf{f}^\epsilon, \quad \mathbf{x} \in D^\epsilon$$

- Need separation of scales,

$$\epsilon = \frac{\ell}{L} \ll 1$$

Multiple Scale Expansion

Field Variables: $\mathbf{u}^\epsilon(\mathbf{x}) = \mathbf{u}^0(\mathbf{x},\mathbf{y}) + \epsilon\mathbf{u}^1(\mathbf{x},\mathbf{y}) + \epsilon^2\mathbf{u}^2(\mathbf{x},\mathbf{y}) \ldots, \quad \mathbf{y} = \epsilon^{-1}\mathbf{x}$

Derivatives: $\partial_{x_j} \mapsto \partial_{x_j} + \epsilon^{-1}\partial_{y_j}$

Expand equations, Match orders of ϵ (Fredholm Alternative)

Fine Scale: Coupled Stokes, Perfectly Periodic Medium

Grain Matrix

$$\nabla_X \sigma^s + \rho_s \mathbf{g} = 0$$
$$\nabla_X \cdot \mathbf{v}^s = 0$$
$$\sigma^s = -p^s I + 2\mu_s e_X(\mathbf{v}^s)$$

Molten Rock

$$\nabla_X \sigma^f + \rho_f \mathbf{g} = 0$$
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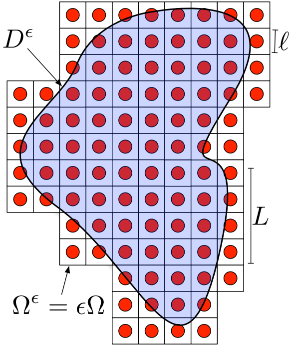
Interface

$$\sigma^s \cdot \mathbf{n} = \sigma^f \cdot \mathbf{n}$$
$$\mathbf{v}^s = \mathbf{v}^f$$



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Closures

- Seek $k_{\text{eff.}} = k_{\text{eff.}}(\phi)$ and $\zeta_{\text{eff.}} = \zeta_{\text{eff.}}(\phi)$; ϕ is a proxy for the microstructure

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Closures

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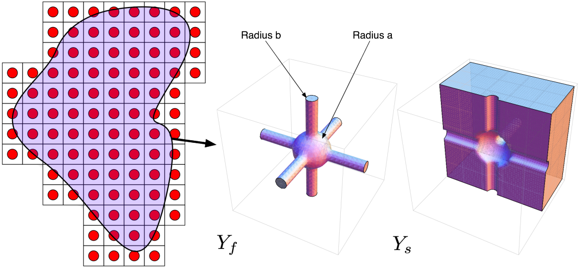
- $\eta_{\text{eff.}}$ is new; macroscopic manifestation of grain scale anisotropy

Computational Closures: Cell Problems

- $k_{\text{eff.}}$, $\zeta_{\text{eff.}}$, $\eta_{\text{eff.}}$ are determined by solving Cell problems, Stokes problems posed on a unit cell



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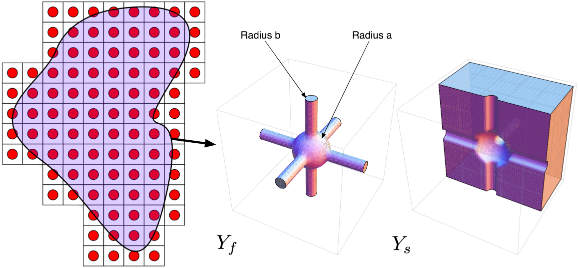

- Bulk Viscosity Cell Problem:

$$\nabla_y \cdot (-\zeta I + 2e_y(\boldsymbol{\xi})) = 0, \quad \nabla_y \cdot \boldsymbol{\xi} = 1, \quad \text{in } Y_s,$$

$$(-\zeta I + 2e_y(\boldsymbol{\xi})) \cdot \mathbf{n} = 0, \quad \text{on } \gamma$$

$$\zeta_{\text{eff.}} = \mu_s \left(\langle \zeta \rangle_s - \tfrac{2}{3}(1 - \phi) \right)$$

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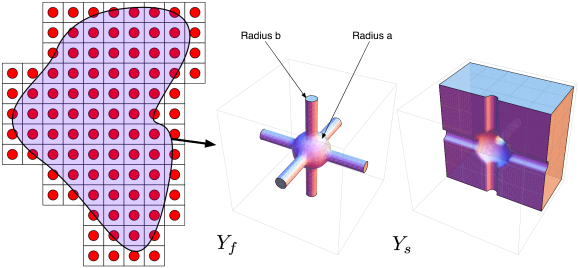

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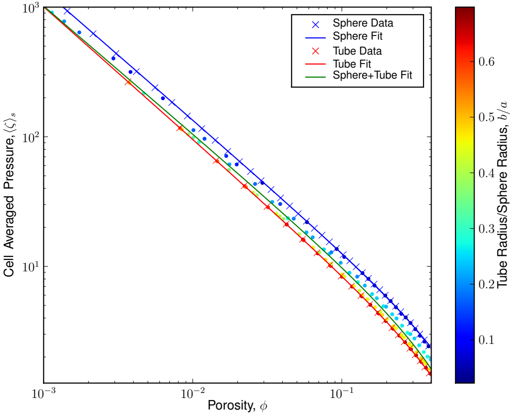
- Solve ensembles (3D FEM); curve fit as a function of ϕ

Computational Results for Bulk Viscosity



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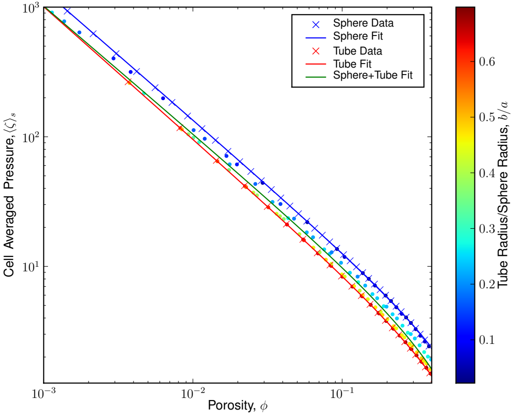

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Tube Domain: $\langle\zeta\rangle_s = \exp(-0.131)\phi^{-1.02}(1-\phi)^{0.884}$

Sphere+Tube Domain: $\langle\zeta\rangle_s = \exp(0.124)\phi^{-0.985}(1-\phi)^{1.09}$

Sphere Domain: $\langle\zeta\rangle_s = \exp(0.301)\phi^{-1.00}(1-\phi)^{0.718}$

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  - For $\phi \sim 1 - 10\%$, two length scales emerge: shear deformation, dilatational deformation:

$$0 = \bar{\rho}\mathbf{g} - \nabla P + \nabla \cdot \left[(1-\phi)\mu_s \left(\nabla \mathbf{V}^s + (\nabla \mathbf{V}^s)^T\right)\right]$$
$$+ \nabla \left[(1-\phi)(\zeta_{\text{eff.}} - \tfrac{2}{3}\mu_s)\nabla \cdot \mathbf{V}^s\right] + \nabla \cdot \left[2\eta^{lm}_{\text{eff.}} e_{lm}(\mathbf{V}^s)\right]$$

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  - Matrix resists $\phi \to 0$

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Remarks on Homogenization

- A self-consistent model of a partial melt can be obtained by multiple scale methods
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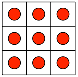
- Closures still require computation

Time Dynamics

- Results assumed a static, periodic, microstructure



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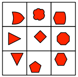

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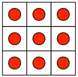

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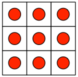

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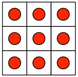

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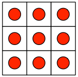

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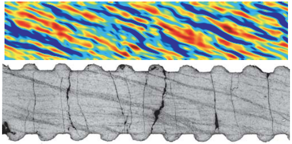
- Dynamic Homogenization
  - Large deformations of a medium–magma & beyond
  - Close constitutive relations of multiphase flow

Complimentary Deformation Mechanism

Pressure Solution/Grain Boundary Diffusion



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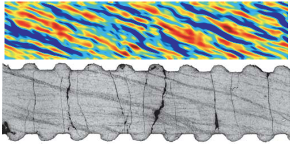

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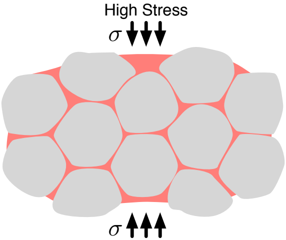

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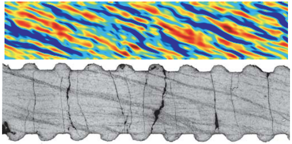

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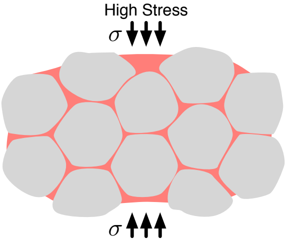

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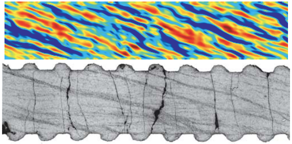

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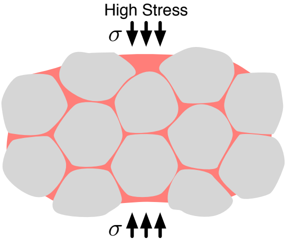

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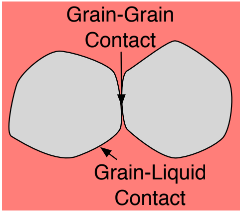
Complimentary Deformation Mechanism, Briefly

Takei & Holtzman Grain Boundary Diffusion



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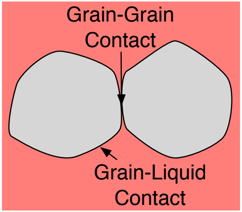

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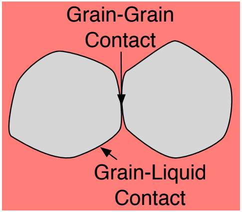

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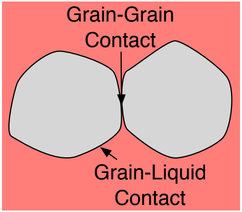

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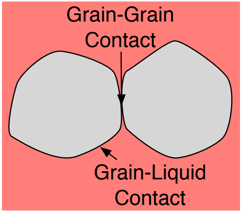

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Acknowledgments

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- `http://www.math.umn.edu/~gsimpson/`