Prove that, for $x \neq 1$, $n$ a nonnegative integer,

$$\sum_{i=0}^{n} x^i = \frac{x^{n+1} - 1}{x - 1}.$$
Similarly, we have alternate notation for the product of several numbers:

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Now that you have nice notation, you could prove, say, that for \( n \geq 1 \),

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(It’s like induction and recursion are somehow related.)
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Examples:

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Instructor: Mike Picollelli
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- The set \( A \) of integers that lie between \(-3\) and \(3\) inclusive, which we can write as

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A = \{ a \mid a \in \mathbb{Z}, -3 \leq a \leq 3 \} = \{-3, -2, -1, 0, 1, 2, 3\}.
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(The $\in$ symbol means “is an element of”.)
- There is exactly one set with no elements at all. It’s called the empty set and written $\emptyset$. 

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**Examples:** $|\emptyset| = 0$, $|\mathbb{N}| = \infty$, and $|\{1, 2, 4, 5\}| = 4$. 